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THE

237

NORMAL

ELEMENTARY ARITHMETIC:

EMBRACING

A COURSE OF EASY AND PROGRESSIVE EXERCISES IN ELEMENTARY WRITTEN ARITHMETIC;

DESIGNED FOR

PRIMARY SCHOOLS, AND PRIMARY CLASSES IN COMMON SCHOOLS, GRADED SCHOOLS, MODEL SCHOOLS, ETC.

REVISED EDITION.

BY

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"The highest science is the greatest simplicity."

PHILADELPHIA:
CHRISTOPHER SOWER CO.,
614 ARCH STREET.
1893.

PREFACE.

The object of this work is to furnish young pupils with an introductory course of *Written Arithmetic*. It is designed as a stepping-stone from the course in primary arithmetic, whether given orally or by use of a text-book, to a complete course in written arithmetic. It is also especially adapted to furnish a complete elementary course in arithmetic for pupils whose time for the study is limited to a very brief course.

The work will be found to possess at least five distinguishing features: 1st, Simplicity; 2d, Gradation; 3d, Practical Character of the Problems; 4th, Variety of Problems; 5th, Educational Character.

SIMPLICITY.—Great care has been taken to make the definitions, explanations, solutions, rules, etc. so simple that the youngest pupils can easily understand them. In doing this, however, the scientific character of the subject has not been sacrificed; for it should ever be remembered that *the highest science is the greatest simplicity;* and, conversely, *the greatest simplicity is the highest science.*

GRADATION.—The gradation of the work will be found one of its most distinctive and valuable features. Great pains have been taken to avoid those sudden transitions from the easy to the difficult for which elementary works are so often criticised. As an example of this feature, see the exercises in Addition, Subtraction, Multiplication, and Division, where the problems are arranged into *classes* and *cases* with respect to their length and difficulty. The same spirit of gradation will be found running through the whole work.

PRACTICAL PROBLEMS.—Arithmetics have been criticised for the abstract and unpractical character of their problems. To avoid this error, I have given a large number of practical problems, drawn from the actual events of life. Among these are Historical, Geographical, and Biographical problems; problems on the battles of the Revolution; farmers', merchants', etc. problems. Such problems not only will add interest to the study of arithmetic, but will present much valuable information to the pupil.

VARIETY OF PROBLEMS.—"Variety is the spice of life," in the school-room as well as out of it. It is a great mistake to keep pupils upon Addition for several months, until they have thoroughly mastered it, then upon Subtraction for a corresponding length of time, and so on for the other fundamental operations. The better way is to give them a fair knowledge of Addition, then take them to Subtraction, and, after they are somewhat familiar with this, give them exercises combining Addition and Subtraction, and thus through the fundamental rules, leaving each subject before the pupil wearies of it, and returning to it again and again, until it is thoroughly mastered. In this manner tiresome monotony is avoided, and the task of the learner rendered interesting and attractive. This is a feature of the present work which it is believed will commend it to intelligent instructors.

EDUCATIONAL CHARACTER.—This work, like the others of the same series, is characterized by an *educational spirit*. It is not a mere collection of problems and rules for the training of a pupil to labor like a machine. The spirit of analysis runs through it, making it *normal* in the broadest sense of the term. Its object is to teach pupils to *think* as well as to *work problems*,—to *develop mind* as well as the power of *computation*.

Cherishing the hope that it may aid teachers in their arduous labors, and become a favorite with the little girls and boys of our common schools, I now intrust it to the decision of a kind and appreciative public.

EDWARD BROOKS.

STATE NORMAL SCHOOL,
May 20, 1865.

NOTE.—In compliance with a general request, I have in the present edition added several pages on the Applications of Percentage and also a section on Practical Measurements, and I have also presented some additional suggestions for *oral instruction* with *concrete illustrations*.

EDWARD BROOKS.

MAY 24, 1888.

CONTENTS.

GENERAL SUGGESTIONS.

1. This book is designed to be put into the hands of young pupils soon after they begin the study of numbers. As soon as pupils can add, subtract, multiply, and divide small numbers they will be prepared for this work.

2. The *Introduction* is not designed to be studied by the pupils, but indicates a course of *Oral Instruction* in the elements of arithmetic. The pupils should be drilled upon such exercises before beginning the study of a text-book on arithmetic; and oral exercises will be found valuable during the entire course, especially upon beginning a new subject.

3. Pupils should solve the problems on the slate or on paper, and also be required to work them out upon the blackboard and explain them. In assigning problems at the board, the same problem may be given to the whole class, or each pupil may receive a different problem. Sometimes one method is preferred, and sometimes the other. The object should be thoroughness and accuracy, and at the same time variety and interest.

4. In many cases two forms of explanation have been given—one a full logical form, the other an abridged mechanical one. The object of the first is to present the reasoning process in full; the object of the second, to give the steps in the method of operation. Though it is important that the pupil should understand the subject as presented in the complete logical form, yet for the ordinary recitation, with young pupils, the abbreviated form may be preferred. With young pupils the primary object is skill and accuracy of operation.

5. Where a pupil has difficulty with a problem owing to its being a little complicated, let the teacher lead him from one step to another by a judicious series of questions, aiding him to analyze the problem, and thus unfold its complexity. This will be much better for the pupil than to work the problem for him. By attention to these suggestions, and to such other points as will occur to the mind of the intelligent teacher, the progress of the pupil will be rapid and thorough.

INTRODUCTION.

SUGGESTION FOR ORAL INSTRUCTION.

THE first lessons in arithmetic should be given orally by the teacher, without the use of a text-book by the pupils. Every teacher should be qualified to give such an oral course, and we present a few suggestions which may be of use to guide young teachers in the work.

COURSE OF INSTRUCTION.

The course of instruction in the elements of arithmetic includes the following subjects:

1. Ideas and Names of Numbers.
2. The Method of Writing Numbers.
3. Elementary Sums and Differences.
4. Elementary Products and Quotients.
5. The Fundamental Operations.
6. The Elements of Fractions.
7. The Elements of Denominate Numbers.

By the *Elementary Sums and Differences* we mean the *sums* of the *addition table* with their corresponding *differences.* By the *Elementary Products and Quotients* we mean the *products* of the *multiplication table* with their corresponding *quotients.* By the *Fundamental Operations* we mean the method of adding, subtracting, multiplying, and dividing larger numbers than those of the tables of addition and multiplication.

One general principle of the methods of teaching these subjects is that in this primary course *they should be more or less combined in instruction.* That is, as soon as pupils have a few ideas of numbers they should begin to write them and to add and subtract them; as soon as they can add and subtract a few numbers they can begin to multiply and divide them, etc. In other words, one subject is not to be completed before another subject is begun, but all are to be harmoniously united in the manner *indicated by the mental growth of the pupil.*

IDEAS AND NAMES OF NUMBERS.

IDEAS.—The child's first lessons in numbers should be given with visible objects. We may use *apples*, *books*, *beads*, *beans*, *lines* and *circles* on the board, etc. The *arithmetical frame* is the most convenient for general purposes. To give ideas of *groups* we can use *bunches of matches*, *toothpicks*, *little sticks*, or *circles with dots* in them.

NAMES.—The names of numbers are taught in connection with the ideas, and both are given in *counting*. In teaching these names, make the pupils first familiar with them as far as *ten*. Then lead them to conceive *ten* as a *group*, and instead of saying *eleven*, *twelve*, etc., teach the child to say *one and ten*, *two and ten*, etc.; *two tens*, *two tens and one*, *two tens and two*, etc. Taught in this way, pupils will have no difficulty in learning to *write numbers*. They can subsequently be permitted to use *eleven*, *twelve*, etc.

THE WRITING OF NUMBERS.

Teach first the meaning of the characters from 1 to 9, and drill the children in making them. Do not then pass immediately to writing *ten*, *eleven*, etc., but show that *four and ten* is expressed thus, 14; *three and ten* thus, 13; *two and ten* thus, 12; *one and ten* thus, 11; and thus lead them to see the necessity of a new character expressing *nothing*, to show that the 1 to denote *ten* (10) is in the second place, and thus introduce the 0.

Taught in this way, children will have no difficulty in understanding the principle of *place-value* in our method of notation. Afterward pass gradually to larger numbers as the pupils are prepared for it. (For more detailed suggestions for teaching pupils to name and write numbers, see page 15.)

ADDITION AND SUBTRACTION.

As soon as a pupil knows a few numbers and their names, he should be led to *unite* and *separate* them. Instruction in these two processes should be given in accordance with the following principles:

1. *Addition and Subtraction should be taught with visible objects.* This principle is founded on the law of mental development, and is evident to every thoughtful teacher.

2. *Addition and Subtraction should be taught together in the first lessons.* This is evident, since the ideas are so intimately related that as soon as the child sees the *sum* of two numbers he has the elements of their *difference*, and can be led to immediately see it. Thus, as

soon as he knows that 2 and 3 are 5, he can see that 5 diminished by 3 is 2, or 5 diminished by 2 is 3.

3. *The pupils must be trained to commit the elementary sums and differences.* In other words, they should be required to commit an addition table, and be able to derive the corresponding differences directly from it. These are the *alphabet of arithmetic*, and must be fixed in the memory. The table should run as far as 9 and 9, or practically as far as 12 and 12.

4. *Pupils should be required to make their own Addition and Subtraction tables.* The frequent making of them will help to fix them in the memory.

EXERCISES IN ADDING AND SUBTRACTING.

1. Pupils should first learn to *increase* and *diminish* by *one*, then by *two*, then by *three*, etc. to *nine;* as, 1 and 1, 2 less 1; 2 and 1, 3 less 1, etc.

2. At first, in writing, we should place the numbers *under one another*, as this is the way we use them in actual practice; afterward we can unite them with the signs, as $2 + 3 = 5$, or $5 - 3 = 2$.

3. Pupils should be taught to *see the sums* at a glance when the numbers are written, just as they know a word the moment they see it.

4. The first written exercises should be similar to the following:

ADDITION.									SUBTRACTION.								
2	2	2	2	2	2	2	2	2	3	4	5	6	7	8	9	10	11
1	2	3	4	5	6	7	8	9	2	2	2	2	2	2	2	2	2
3	3	3	3	3	3	3	3	3	4	5	6	7	8	9	10	11	12
1	2	3	4	5	6	7	8	9	3	3	3	3	3	3	3	3	3
4	4	4	4	4	4	4	4	4	5	6	7	8	9	10	11	12	13
1	2	3	4	5	6	7	8	9	4	4	4	4	4	4	4	4	4

Exercises like the above should be continued as far as 9 and 9. In these the pupils will add or subtract from below upward; thus, 1 and 2 are 3, 2 and 2 are 4, etc. Also, 2 from 3 leaves 1, 2 from 4 leaves 2, etc.

5. With these, also, exercises like the following should be used, in which, the signs being explained, pupils are required to fill out the vacant places:

$1+2=$	$3-2=$	$1+3=$	$4-3=$	$1+4=$	$5-4=$
$2+2=$	$4-2=$	$2+3=$	$5-3=$	$2+4=$	$6-4=$
$3+2=$	$5-2=$	$4+3=$	$6-3=$	$3+4=$	$7-4=$
etc.	etc.	etc.	etc.	etc.	etc.

6. After the pupils know the sums to 9 plus 9, and the differences as far as 18 minus 9, they should be taught to add and subtract numbers from 1 to 9 to larger numbers, expressed in tens and units, as in the following exercises:

ADDITION.							SUBTRACTION.						
10	11	12	13	23	32	41	12	14	16	18	28	29	29
2	3	4	5	6	7	8	2	3	4	5	6	8	9

7. Following these exercises they should be taught to add and subtract numbers not exceeding 9 to numbers not exceeding 99, in which they must "carry" and "borrow," as in the following:

ADDITION.							SUBTRACTION.						
24	27	23	25	26	28	22	29	33	31	32	30	31	31
5	6	8	7	4	3	9	5	6	8	7	6	3	9
32	36	35	37	34	36	38	36	39	43	42	41	39	40
4	3	8	5	7	6	2	4	3	8	5	7	6	2

After the three classes of exercises presented in Arts. 4, 6, and 7, pupils will be prepared for the general methods of Addition and Subtraction.

MULTIPLICATION AND DIVISION.

After the pupil is familiar with a few of the elementary sums and differences, he can begin to learn the elements of Multiplication and Division. Instruction in these processes should be given in accordance with the following principles:

1. *Multiplication should be taught as concise Addition.* Thus, the pupil should be taught that 2 times 3 are 6, because $3+3$ are 6. Multiplication will thus be conceived as a derivative of Addition.

2. *Division should be taught as reverse Multiplication.* Thus, it should be shown that 6 contains 3 two times, because two 3's are 6. In this way the quotients can be immediately derived from the products.

3. After learning to make the table in the ordinary way, he should be led to see that *each product* is equal to *the previous product increased by the multiplier.* Thus, 4 times 5 are 20, and 4 times 6 are 4 more than 20, or 24; 4 times 7 are 4 more than 24, or 28, etc.

4. *Pupils should be taught to construct the multiplication table for themselves.* This they can readily do if they have been taught that Multiplication is concise Addition.

5. *Multiplication and Division should be taught together in the first lessons.* This is evident from the intimate relation of the ideas. Thus, as soon as the pupil sees that 2 times 3 are 6 he is ready to see that 6 equals 2 *threes* or contains 3 *two* times.

6. *Pupils should be required to commit the elementary products and quotients.* That is, they should commit the multiplication table and be able to derive the quotients directly from it.

EXERCISES IN MULTIPLYING AND DIVIDING.

1. Pupils should first learn the products and quotients of 2 times, then of 3 times, 4 times, etc., as far as 9 times, or, in practice, 12 times.

2. Exercises similar to the following may be used until the pupils have committed the elementary products up to 12 times:

$2 \times 1 =$	$2 \div 1 =$	$3 \times 1 =$	$3 \div 1 =$
$2 \times 2 =$	$4 \div 2 =$	$3 \times 2 =$	$6 \div 2 =$
$2 \times 3 =$	$6 \div 3 =$	$3 \times 3 =$	$9 \div 3 =$
etc.	etc.	etc.	etc.
$2 \times 12 =$	$24 \div 12 =$	$3 \times 12 =$	$36 \div 12 =$

3. Exercises similar to the following may also be used in connection with the above, requiring pupils to fill out the blanks:

$4 \times \quad = 4$	$4 \div \quad = 4$	$5 \times \quad = 5$	$5 \div \quad = 5$
$4 \times \quad = 8$	$8 \div \quad = 4$	$5 \times \quad = 10$	$10 \div \quad = 5$
$4 \times \quad = 12$	$12 \div \quad = 4$	$5 \times \quad = 15$	$15 \div \quad = 5$
etc.	etc.	etc.	etc.

4. In order to *commit the multiplication table,* pupils should *make* it, *write* it, *study* it, *recite* it, and even *sing* it.

THE FUNDAMENTAL OPERATIONS.

In connection with these exercises in learning the elementary sums, differences, products, and quotients, the pupils should be trained to apply these results to the Arabic method of notation—that is, to written arithmetic proper.

In Addition they should be taught "to carry" when the sum exceeds nine; in Subtraction, "to borrow" when a term of the subtrahend exceeds the term in the minuend; in Multiplication, to multiply by the different terms of the multiplier; and in Division, the methods of Short and Long Division.

Pupils must be trained until they become expert in these processes. For this instruction a text-book with examples will be found convenient. The methods of teaching are given in this book under Addition, Subtraction, Multiplication, and Division.

In this instruction pupils may be taught the reason for the operations along with the operation, or, what some teachers think is preferable with beginners, *we may teach the method first and the reason for it afterward.*

THE ELEMENTS OF FRACTIONS.

The Elements of Fractions should be introduced in connection with the previous instruction, and be carried on along with it.

These elements should be presented *in the concrete* by the use of *objects*, *lines*, and *circles* on the *board*, etc.

For the method of teaching the elements of fractions, see page 96 of this book.

THE ELEMENTS OF DENOMINATE NUMBERS.

The Elements of Denominate Numbers should be introduced in connection with the instruction in the elements of Multiplication and Division. In this way pupils become familiar with the principal measures before they take up the subject formally in the book.

These elements should be presented *concretely* by having the different *units in the school-room*, when possible, for the pupils to see and handle. They should include the *foot*, *inch*, and *yard*; the *pint*, *quart*, and *gallon*; the *pound* and *ounce*; the *pint*, *quart*, and *peck* (dry measure); and the *day*, *hour*, *week*, *year*, and *month*.

Little problems in reducing from one unit to another can be given in connection with Multiplication and Division; as, how many feet in 2 yards? yards in 6 feet? pints in 2 quarts? quarts in 8 pints? etc., etc. For other suggestions on the subject, see page 140.

OTHER SUBJECTS IN ARITHMETIC.

Besides the subjects named as included in the elements of arithmetic, the present work embraces Factoring, Common Multiples and Divisors, Decimals, and Percentage. Suggestions for oral instruction in these subjects will be found in connection with the treatment of them.

THE GRUBE METHOD.

The method now presented for primary instruction in arithmetic is what may be called the *Normal Method.* There is another method of teaching the elementary sums, differences, products, and quotients, called the *Grube Method.* Every intelligent teacher should understand this method, though he may not desire to use it.

The principles of the Grube method, stated briefly, are as follows:

1. *Addition, Subtraction, Multiplication, and Division are taught together from the beginning.* Thus, if 2 is the subject of the lesson, the child is taught that $1 + 1 = 2$, $2 - 1 = 1$, $2 - 2 = 0$, $2 \times 1 = 2$, $1 \times 2 = 2$, $2 \div 1 = 2$, $2 \div 2 = 1$.

2. *Each number, taken in order, is made the basis of the lesson.* Thus, after applying all the four operations to 2, the same thing is done with 3, then with 4, 5, 6, etc., continuing in this way until we reach 100 or 144.

3. Advocates of this method say *the course of instruction the first year should go as far as* 10, the second year as far as 20, the third year as far as 100.

4. All this instruction is to be given with objects, as already explained in the Normal Method.

A few of the differences of the two methods seem to be as follows.

1. The Normal Method combines adding and subtracting from the beginning; the Grube Method combines adding, subtracting, multiplying, and dividing from the beginning.

2. The Normal Method makes the *operations* with numbers the basis of instruction; the Grube Method makes the *numbers* themselves the basis.

3. The Normal Method aims at *skill in the use of numbers;* the Grube Method makes what its advocates call the *comprehension of numbers* the leading object.

4. The Normal Method covers the entire ground of primary instruction in arithmetic; the Grube Method does not extend beyond learning the elementary sums, differences, products, and quotients.

EXERCISES IN THE METHOD.

For those who wish to use this method in teaching the elementary sums, differences, etc., we present the following exercises on numbers as far as 6. The exercises beyond 6 can be readily supplied by any teacher desiring to use this method.

The Number 2.—The exercises on *Two* are as follows:
$2=1+1$; $2-1=2$; $2-2=0$.
$2=2\times1$; $2=1\times2$; $2\div1=2$; $2\div2=1$.

The Number 3.—The exercises on *Three* are as follows:
$3=1+1+1$; $3=2+1$; $3=1+2$; $3-1=2$; $3-2=1$; $3-3=0$.
$3=3\times1$; $3=1\times3$; $3\div1=3$; $3\div3=1$.

The Number 4.—The exercises on *Four* are as follows:
$4=1+1+$ etc.; $4=3+1$; $4=2+2$; $4-1=3$; $4-2=2$; $4-3=1$, etc.
$4=4\times1$; $4=1\times4$; $4=2\times2$; $4\div1=4$; $4\div4=1$; $4\div2=2$.

The Number 5.—The exercises on *Five* are as follows:
$5=1+1+$ etc.; $5=4+1$; $5=3+2$; $5=2+3$; $5=1+4$; $5-1=4$; $5-2=3$; $5-3=2$; $5-4=1$; $5-5=0$.
$5=5\times1$; $5=1\times5$; $5\div1=5$; $5\div5=1$.

The Number 6.—The exercises on *Six* are as follows:
$6=1+1+$ etc.; $6=5+1$; $6=4+2$; $6=3+3$; $6=2+4$, etc.; $6-1=5$; $6-2=4$; $6-3=3$; $6-4=2$; $6-5=1$, etc.
$6=6\times1$; $6=1\times6$; $6=3\times2$; $6=2\times3$.
$6\div1=6$; $6\div6=1$; $6\div2=3$; $6\div3=2$.

The Number 7.—The exercises on *Seven* are as follows:
$7=1+1+$ etc.; $7=6+1$; $7=5+2$; $7=4+3$; $7=3+4$, etc.; $7-1=6$; $7-2=5$; $7-3=4$; $7-4=3$; $7-5=2$, etc.
$7=7\times1$; $7=1\times7$; $7\div7=1$; $7\div1=7$.

The Number 8.—The exercises on *Eight* are as follows:
$8=1+1+$ etc.; $8=7+1$; $8=6+2$; $8=5+3$; $8=4+4$; $8-1=7$; $8-2=6$; $8-3=5$; $8-4=4$; $8-3=5$, etc.
$8=8\times1$; $8=1\times8$; $8=4\times2$; $8=2\times4$; $8=2\times2\times2$.
$8\div1=8$; $8\div8=1$; $8\div2=4$; $8\div4=2$.

All the other numbers to 100 or 144 are to be treated in a similar manner. Teachers who desire to use this method can readily make out these exercises for their pupils.

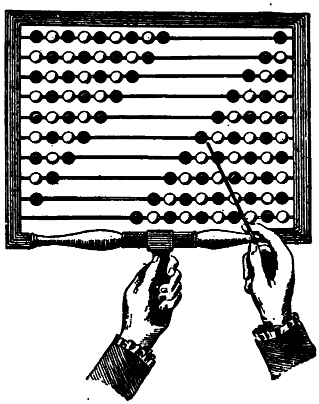
INTRODUCTION TO NUMERATION AND NOTATION.

(SUGGESTIONS TO THE TEACHER.)

NAMING NUMBERS.

1. With the numeral frame or with objects, have pupils count from *one* to *ten*.

2. Then with ten balls on the upper wire bring over successively other balls on the second wire, and count *one and ten*, *two and ten*, etc., to *two tens*.



3. Then with ten balls on the first and the second wire bring over balls on the third wire, and count *two tens and one*, *two tens and two*, to *three tens*.

4. Proceed in this manner as far as *ten tens*, and teach that *ten tens* is called a *hundred*.

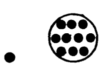
5. Then lead the pupil to see that *three and ten*, *four and ten*, etc., are abbreviated into *thirteen*, *fourteen*, etc.

6. Similarly show that *two tens*, *three tens*, *four tens*, etc., are abbreviated into *twenty* (twain-tens), *thirty*, *forty*, etc.

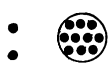
7. Lead then to the names of numbers beyond one hundred as far as is thought desirable.

8. Lead pupils to see clearly that when we have *ten* things we make *a group* of them and count them as *one of the group*.

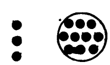
9. Illustrate as below with *dots* representing units, and circles containing ten units, representing tens, counting *one and ten*, *two and ten*, etc.



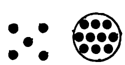
One and ten.



Two and ten.

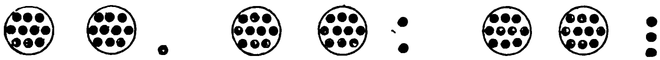


Three and ten.



Five and ten.

10. Illustrate also as below in counting *two tens and one, two tens and two,* etc.



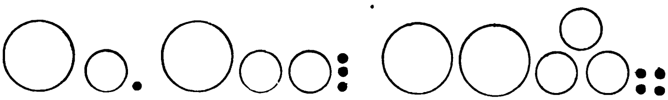
Two tens and one. Two tens and two. Two tens and three.

11. Let the dots be omitted from the circle and use the circle to represent tens, and count as follows:

Two tens and four. Two tens and five. Three tens and six.

12. Illustrate also that when we have *ten* of these tens we call it *hundred.* Represent the *hundred* by a larger circle, *tens* by a smaller circle, and *ones* by dots.

13. Have exercises in naming numbers represented by these characters, as follows:



One hundred, 1 ten, and 1. One hundred, 2 tens, and 3. Two hundred, 3 tens, and 4.

NOTE.—The ease or difficulty with which pupils learn to *write* numbers depends almost entirely on how they have been taught to *name* numbers. When pupils are taught to count by saying *one and ten, two and ten,* etc., *two tens and one, two tens and two,* etc., they will experience no difficulty in learning to write numbers.

WRITING NUMBERS.

1. In teaching pupils to write numbers, teach first the nine digits, *1, 2, 3, 4, 5, 6, 7, 8, 9,* and drill pupils until they can make them and know what they denote.

2. Have them solve little problems with these digits in addition, subtraction, etc., until they are entirely familiar with them.

3. In teaching to write numbers beyond 9, begin with *fourteen,* or *four and ten,* and lead them to see that we write "4 and 1 ten" thus, 14, the 4 denoting *four* and the 1 denoting the 1 *ten.*

4. Next lead them to see that we write *five and ten* (fifteen), *three and ten* (thirteen), *two and ten* (twelve), *one and ten* (eleven), in a similar manner.

5. Then lead them to see that to write *one ten* we need a character to take the place of the 4, 3, 2, and 1 in 14, 13, 12, 11, to show that the 1 ten is in the *second place,* and we might use a *square* first to denote no units, as in the margin, and, then rounding the corners, reach the *cipher.*

6. The pupil can then be easily led to see how to express 2 *tens,* 3 *tens,* etc., and will have no difficulty in expressing numbers as far as 99.

7. The method of expressing *one hundred* may be taught in a similar manner. Taught in this way, pupils will find very little difficulty with notation.

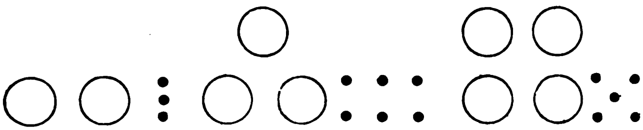
1□
One ten.

10
One ten.

20
Two tens.

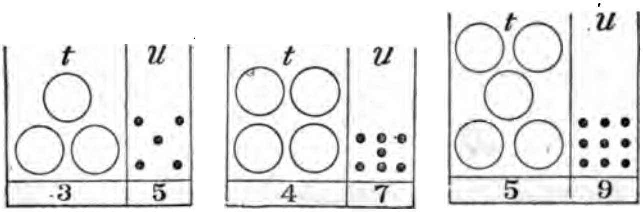
30
Three tens.

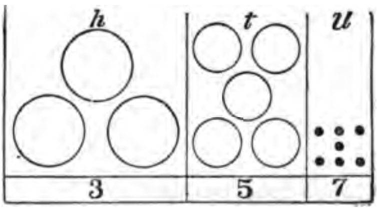
8. The subject may be illustrated with the *dots* and *circles* used in naming numbers. Thus:



2 3 3 6 4 5

9. We can also draw vertical lines and use these dots and circles to represent the number of *units, tens,* and *hundreds.* Thus:





10. Vertical lines may be drawn and headed *h*, *t*, *u*, and pupils be required to put figures in their proper places. Thus express *twenty-five, thirty-six, forty; three hundred and twenty-five, four hundred and sixty-eight, five hundred and seven*, etc.

t	*u*
2	5
3	6
4	0

h	*t*	*u*
3	2	5
4	6	8
5	0	7

h	*t*	*u*
5	6	0
6	0	9
5	7	0

T	*h*	*t*	*u*
2	4	3	8
3	0	7	5
6	4	0	3

NOTE.—Great variety can be given to these illustrations, and they will afford a pleasant exercise to the ingenuity of the teacher as well as to that of the children.

Remember, however, that these concrete illustrations are merely stepping-stones to the subject. The pupil must be led to understand these subjects in the abstract, independently of the illustrations, or he does not comprehend them fully.

Visible methods similar to those suggested were used by semi-savage tribes in the early history of civilization, but the invention of the Arabic or Hindoo arithmetic has lifted mankind above these concrete symbols. Pupils must be taught to *think* arithmetic rather than to *see* arithmetic.

For early methods which modern arithmetic has rendered obsolete, see Brooks's *Philosophy of Arithmetic.*

THE

NORMAL

ELEMENTARY ARITHMETIC.

SECTION I.

1. A ***Unit*** is a single thing, as a *book*, *pen*, *apple*.

2. A ***Number*** is one or more things, as *two* books, *three* pens.

3. ***Arithmetic*** is the science of numbers and the art of using them.

4. ***Mental Arithmetic*** is the solving of problems without the aid of written characters.

5. ***Written Arithmetic*** is the solving of problems with written characters.

6. In studying arithmetic we first learn to *name* numbers, and then learn to *write* them.

NUMERATION.

7. ***Numeration*** is the art of naming numbers, and of reading them when they are written.

8. We will give the names of some of the numbers up to one hundred.

one ;	**six ;**
two ;	**seven ;**
three ;	**eight ;**
four ;	**nine ;**
five ;	**ten ;**

eleven, or one and ten;
twelve, or two and ten;
thirteen, or three and ten;
fourteen, or four and ten;
fifteen, or five and ten;
sixteen, or six and ten;
seventeen, or seven and ten;
eighteen, or eight and ten;
nineteen, or nine and ten;
twenty, or two tens;
twenty-one, or two tens and one;
twenty-two, or two tens and two;
thirty, or three tens;
thirty-one, or three tens and one;
forty, or four tens;
forty-one, or four tens and one;
fifty, or five tens;
fifty-one, or five tens and one;
sixty, or six tens;
sixty-one, or six tens and one;
seventy, or seven tens;
etc., etc.

NOTE.—The pupil should be drilled upon these equivalent forms of naming numbers, as a preparation for Notation. The teacher or pupil may fill out the omissions.

ARABIC NOTATION.

9. ***Notation*** is the **art of writing numbers.**

10. ***Figures.***—In writing numbers we use the following ten characters, called *figures.*

1	expresses	one.	6	expresses	six.
2	"	two.	7	"	seven.
3	"	three.	8	"	eight.
4	"	four.	9	"	nine.
5	"	five.	0	"	naught.

11. ***Combination.***—By these figures and their combinations all numbers can be expressed.

The method of combining them is as follows:—

1st. *A figure standing alone expresses units or ones.*

2d. *When two figures are together, the one in the first place at the right expresses units, the one in the second place expresses tens.*

3d. *A figure in the third place expresses hundreds, in the fourth place thousands*, etc.

12. Thus, in 25, the 2 expresses 2 *tens*, and the 5 expresses 5 *units*. We will illustrate this by the following table.

10 is one ten.
20 " two tens.
30 " three tens.
40 " four tens.
50 " five tens.
60 " six tens.
70 " seven tens.
80 " eight tens.
90 " nine tens.
100 " one hundred.

11 is 1 ten and one.
12 " 1 ten and two.
23 " 2 tens and three
34 " 3 tens and four.
47 " 4 tens and seven.
58 " 5 tens and eight.
65 " 6 tens and five.
79 " 7 tens and nine.
86 " 8 tens and six.
105 " 1 hundred and five.

13. The pupils will now write and read the following numbers:

13	24	33	41	57	74
15	26	36	43	60	76
17	27	38	45	68	82
19	29	39	50	63	95
21	30	40	52	70	126

14. The pupils will now learn the names of the first twelve places, as represented in the following

NUMERATION TABLE.

NAMES.	Hundred-billions	Ten-billions.	Billions.	Hundred-millions	Ten-millions.	Millions.	Hundred-thousands.	Ten-thousands.	Thousands.	Hundreds.	Tens.	Units.
	4	4	4	4	4	4	4	4	4	4	4	4
PLACES.	12th,	11th,	10th,	9th,	8th,	7th,	6th,	5th,	4th,	3d,	2d,	1st.
PERIODS.	4th.			3d			2d.			1st.		

15. The pupil should now be drilled upon questions similar to the following.

Required the names of the following places:

1. First.
2. Third.
3. Fourth.
4. Second.
5. Fifth.
6. Sixth.
7. Eighth.
8. Seventh.
9. Ninth.

Required the places of the following:

1. Tens.
2. Hundreds.
3. Ten-thousands.
4. Thousands.
5. Millions.
6. Hundred-thousands.

16. *Periods.*—For convenience in writing and reading numbers, we arrange the figures in periods of three places each, as shown by the table.

The first three places make the *first* or *units*, *period*, the second three places make the *second* or *thousands*, *period*, etc.

Required the period and place of the following:

1. Hundreds.
2. Thousands.
3. Millions.
4. Ten-thousands.
5. Tens.
6. Ten-millions.
7. Hundred-thousands.
8. Hundred-millions.

17. The combination of figures to express a number forms a *numerical word.* Thus, 25 is the numerical word which means the same as *twenty-five.* These numerical words may be analyzed.

PROBLEMS.

1. Analyze the numerical word 324.

ANALYSIS.—The 4 represents *four units*, the 2 represents *two tens*, the 3 represents *three hundreds:* hence the numerical word is *three hundred and twenty-four.*

Analyze the following:

2. 426	4. 652	6. 853	8. 395	10. 1234	12. 43762
3. 357	5. 785	7. 687	9. 785	11. 5678	13. 85967

18. We will now give some exercises in Numeration and Notation.

RULE FOR NUMERATION.

I. *Begin at the right hand, and separate the numerical word into periods of three figures each.*

II. *Then begin at the left hand, and read each period as if it stood alone, giving the name of each period except the last.*

1. What number is expressed by 3254789?

SOLUTION.—We separate the numerical word into periods of three figures each, as in the margin. The third period is 3 millions, the second period is 254 thousands, the first is 789; hence the number is 3 millions, 254 thousands, 789.

OPERATION.
3,254,789

Read the following:

2.	2384	6.	64327	10.	6321456
3.	7428	7.	52105	11.	78535217
4.	6321	8.	43246	12.	852106721
5.	8357	9.	785625	13.	12345678935

19. Having learned to read numerical words, the pupils are now prepared to write them. From the principles we have given we derive the following

RULE FOR NOTATION.

Begin at the left, and write each figure in order towards the right, giving each figure its proper place, and filling the vacant places with ciphers.

1. Write the number four thousand three hundred and seven.

SOLUTION.—We write the 4 in thousands' place, the 3 in hundreds' place, the 7 in units' place, and, since there are no tens, we write a naught in tens' place.

OPERATION.
4307

Write the following in figures:

2. Three thousand and seventy-five. Ans. 3075
3. Five thousand six hundred and fifty.
4. Seven thousand eight hundred and four.
5. Twenty-three thousand four hundred and ninety.
6. Twenty-five thousand three hundred and seven.
7. Two hundred and six thousand four hundred and six.
8. Four hundred and eighty-six thousand nine hundred and eight.
9. Seven hundred and forty-three thousand four hundred and ninety.
10. Two millions, three hundred thousand four hundred and eighty.
11. Four millions, five hundred and six thousand and twenty-five.
12. Six billions, six millions, six thousands, six hundred and six.

REMARK.—Pupils should be drilled in exercises like those given, until they can read and write numbers readily.

ROMAN NOTATION.

The Roman Method of Notation employs seven letters of the Roman alphabet. Thus, I represents *one;* V, *five;* X, *ten;* L, *fifty;* C, *one hundred;* D, *five hundred;* M, *one thousand.*

To express other numbers these characters are combined according to the following principles:—

1. *Every time a letter is repeated its value is repeated.*
2. *When a letter is placed before one of greater value, the* DIFFERENCE *of their values is the number represented.*
3. *When a letter is placed after one of a greater value, the* SUM *of their values is the number represented.*

4. *A dash placed over a letter increases its value a thousand fold. Thus,* $\overline{\text{VII}}$ *denotes seven thousand.*

ROMAN TABLE.

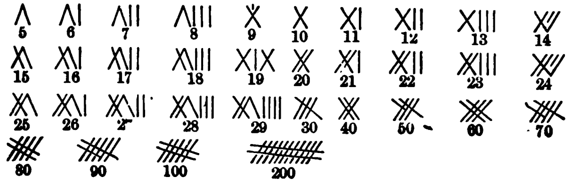
I	One.	XXX	Thirty.
II	Two.	XL	Forty.
III	Three.	L	Fifty.
IV	Four.	LX	Sixty.
V	Five.	LXX	Seventy.
VI	Six.	XC	Ninety.
VII	Seven.	C	One hundred.
VIII	Eight.	CC	Two hundred.
IX	Nine.	D	Five hundred.
X	Ten.	DC	Six hundred.
XI	Eleven.	DCCCC	Nine hundred.
XIV	Fourteen.	M	One thousand.
XV	Fifteen.	MM	Two thousand.
XIX	Nineteen.	MCLX	One thousand one hundred and sixty
XX	Twenty	MDCCCLIX	1859.

The Roman Method is named after the Romans, who invented and used it. It is now employed to denote the chapters and sections of books, pages of preface and introduction, and in other places for prominence and distinction.

LUMBERMEN'S NOTATION.

(Not to be studied unless otherwise directed.)

Lumbermen in marking lumber employ a modification of the Roman Method of Notation. The first four characters are like the Roman; the others are as follows:



INTRODUCTION TO ADDITION AND SUBTRACTION.

If pupils are not already familiar with the *elementary sums* and *differences*, they should be drilled on exercises similar to the following. With beginners we should use the numeral frame, beans, marbles, lines on the slate or blackboard, etc.

Lead the pupil to see the relation of the *difference* to the *sum*, so that if he *knows the sum* he can immediately *derive the difference.*

1. How many are 1 and 2? 2 and 2? 4 and 2? 3 and 2? 5 and 2? 7 and 2? 6 and 2? 8 and 2? 9 and 2? 10 and 2?

2. How many are 3 less 2? 5 less 2? 4 less 2? 6 less 2? 8 less 2? 7 less 2? 9 less 2? 8 less 2? 10 less 2?

3. How many are 1 and 3? 2 and 3? 3 and 3? 5 and 3? 4 and 3? 7 and 3? 6 and 3? 8 and 3? 9 and 3? 10 and 3?

4. How many are 4 less 3? 5 less 3? 7 less 3? 6 less 3? 8 less 3? 10 less 3? 9 less 3? 11 less 3? 12 less 3?

5. How many are $1+4$? $2+4$? $4+4$? $3+4$? $5+4$? $7+4$? $6+4$? $8+4$? $9+4$? $10+4$?

6. How many are $5-4$? $7-4$? $6-4$? $8-4$? $10-4$? $9-4$? $11-4$? $13-4$? $12-4$?

NOTE.—Have pupils give the results of the following exercises orally, and also write them with the sign of equality; thus, $1+5=6$.

7. How many are

$1+5$?	$3+5$?	$2+5$?	$4+5$?	$6+5$?	$5+5$?
$6-5$?	$8-5$?	$7-5$?	$9-5$?	$11-5$?	$10-5$?
$8+5$?	$7+5$?	$9+5$?	$10+5$?	$11+5$?	$12+5$?
$13-5$?	$12-5$?	$14-5$?	$15-5$?	$16-5$?	$17-5$?

8. How many are

$1+6$?	$3+6$?	$2+6$?	$4+6$?	$6+6$?	$5+6$?
$7-6$?	$9-6$?	$8-6$?	$10-6$?	$12-6$?	$11-6$?
$7+6$?	$9+6$?	$8+6$?	$10+6$?	$11+6$?	$12+6$?
$13-6$?	$15-6$?	$14-6$?	$16-6$?	$17-6$?	$18-6$?

9. How many are

$1+7$?	$3+7$?	$4+7$?	$2+7$?	$5+7$?	$7+7$?
$8-7$?	$10-7$?	$11-7$?	$9-7$?	$12-7$?	$14-7$?
$6+7$?	$8+7$?	$10+7$?	$9+7$?	$11+7$?	$12+7$?
$13-7$?	$15-7$?	$17-7$?	$16-7$?	$18-7$?	$19-7$?

10. How many are

$1+8$?	$3+8$?	$2+8$?	$4+8$?	$6+8$?	$5+8$?
$9-8$?	$11-8$?	$10-8$?	$12-8$?	$14-8$?	$13-8$?
$6+8$?	$8+8$?	$10+8$?	$9+8$?	$11+8$?	$12+8$?
$14-8$?	$16-8$?	$18-8$?	$17-8$?	$19-8$?	$20-8$?

11. How many are

$1+9$?	$3+9$?	$2+9$?	$4+9$?	$6+9$?	$5+9$?
$10-9$?	$12-9$?	$11-9$?	$13-9$?	$15-9$?	$14-9$?
$7+9$?	$9+9$?	$8+9$?	$10+9$?	$11+9$?	$12+9$?
$16-9$?	$18-9$?	$17-9$?	$19-9$?	$20-9$?	$21-9$?

NOTE.—These exercises contain all the *sums* and *differences* that need be committed to memory. Pupils should then be taught to apply these to numbers including tens, as follows:

12. Add and also subtract the following:

12	13	15	20	18	21	23	25	30	31
2	2	2	2	2	2	2	2	2	2

13. Add and also subtract the following:

14	13	15	18	19	21	25	27	29	34
3	3	3	3	3	3	3	3	3	3

14. Add and also subtract the following:

14	16	18	20	19	22	26	28	32	38
4	4	4	4	4	4	4	4	4	4

15. Add and also subtract the following:

15	14	17	19	20	24	26	28	32	36
5	5	5	5	5	5	5	5	5	5

16. Add and also subtract the following:

14	17	19	20	23	25	28	32	38	42
6	6	6	6	6	6	6	6	6	6

17. Add and also subtract the following:

20	22	25	26	28	31	34	39	40	48
7	7	7	7	7	7	7	7	7	7

18. Add and also subtract the following:

20	21	24	26	28	32	35	40	43	49
8	8	8	8	8	8	8	8	8	8

19. Add and also subtract the following:

20	22	24	26	23	35	36	27	28	39
9	9	9	9	9	9	9	9	9	9

SECTION II.

ADDITION.

20. ***Addition*** is the process of finding the *sum* of two or more numbers.

21. The ***Sum*** is a number which contains as many units as the numbers added.

22. The ***sign of Addition*** is $+$, and is read *plus*. The ***sign of Equality*** is $=$, and is read *equals*, or *equal to*. Thus, $4 + 5 = 9$, is read 4 *plus* 5 *equals* 9.

CASE I.

23. To add when the sum of a column is not more than nine of that column.

24. *CLASS I.—Problems of one column.*

1. What is the sum of 2, 3, and 4?

SOLUTION.—We write the numbers one under another and begin at the bottom to add. 4 and 3 are 7 and 2 are 9. Hence the sum is *nine*.

OPERATION.

$$\begin{array}{r} 2 \\ 3 \\ \underline{4} \\ 9 \end{array}$$

EXAMPLES FOR PRACTICE.

(2.)	(3.)	(4.)	(5.)	(6.)	(7.)	(8.)
2	4	6	1	7	3	1
1	1	0	5	0	4	2
3	3	3	3	2	2	6
6						

(9.)	(10.)	(11.)	(12.)	(13.)	(14.)	(15.)	(16.)	(17.)
3	2	4	6	2	3	5	5	3
5	1	2	0	1	4	2	1	2
1	4	3	2	6	2	2	3	4

18. What is the sum of 2, 0, 3, 4?
19. What is the sum of 3, 1, 0, 2, 3?
20. What is the sum of 2, 2, 3, 0, 1?
21. What is the sum of 4, 1, 0, 2, 1?
22. What is the sum of 3, 0, 2, 0, 1, 3?

25. *CLASS II.—Problems of more than one column.*

1. What is the sum of 21, 32, 43?

OPERATION.
21
32
43
96

SOLUTION.—We write the numbers so that units stand under units, and tens under tens, and begin at the right to add. Adding the units, we have 3 and 2 are 5, and 1 are 6, which we write in units' place. Adding the tens, we have 4 and 3 are 7 and 2 are 9, which we write in tens' place. Hence the sum is 96.

NOTE.—Have the pupils solve by merely naming the successive sums; as "5, 6," "7, 9."

EXAMPLES FOR PRACTICE.

(2.)	(3.)	(4.)	(5.)	(6.)
31	20	34	12	15
23	14	20	23	40
24	25	15	54	34
78				

(7.)	(8.)	(9.)	(10.)	(11.)
121	214	610	234	361
213	312	156	432	215
432	153	213	123	123
766				

(12.)	(13.)	(14.)	(15.)	(16.)
612	314	712	416	201
105	212	150	141	305
271	271	137	222	281

(17.)	18.)	(19.)	(20.)	(21.)
2021	5234	6141	7124	6214
3514	1321	1213	1321	322
2361	2141	2032	2042	1211

(22.)	(23.)	(24.)	(25.)	(26.)
34123	41210	50273	1234	23071
14310	13025	17202	4012	12303
20341	21613	21310	3701	20413
11111	12030	10101	1020	21210

Required the sum

27. Of 2031, 1234, 3122, and 1010.
28. Of 1207, 3040, 2430, and 2112.
29. Of 2051, 3027, 1500, and 1320.
30. Of 21021, 2712, 12032, 102, and 21.
31. Of 12201, 23021, 2142, and 12012.

CASE II.

26. To add when the sum of any column is more than nine.

27. *CLASS I.—Problems of one column.*

1. What is the sum of 7, 6, and 8?

SOLUTION.—We write the numbers under one another, and begin at the bottom to add. 8 and 6 are 14 and 7 are 21. We place the 1 under the column, and the 2 in tens' place.

OPERATION.

$$\begin{array}{r} 7 \\ 6 \\ \underline{8} \\ 21 \end{array} \text{ Ans.}$$

EXAMPLES FOR PRACTICE.

(2.)	(3.)	(4.)	(5.)	(6.)	(7.)
8	7	8	6	8	3
4	2	2	3	2	9
6	5	7	7	6	7
13					

(8.)	(9.)	(10.)	(11.)	(12.)	(13.)
5	7	6	4	7	6
4	3	3	3	4	7
3	8	8	6	7	3
2	5	7	5	5	8

(14.)	(15.)	(16.)	(17.)	(18.)	(19.)
8	3	7	2	4	7
7	7	1	0	5	8
3	2	3	7	6	9
2	5	5	8	7	6
4	5	6	9	8	5

Required the sum

20. Of 6, 7, 5, 3, 2, and 4.
21. Of 3, 2, 7, 4, 6, and 7.
22. Of 3, 4, 5, 6, 7, and 8.
23. Of 4, 5, 6, 2, 3, and 5.
24. Of 2, 7, 3, 1, 4, and 6.
25. Of 3, 5, 4, 3, 2, and 4.
26. Of 3, 6, 7, 2, 1, and 4.
27. Of 1, 3, 2, 7, 8, and 5.
28. Of 8, 2, 4, 6, 5, and 6.
29. Of 3, 6, 5, 3, 7, and 2.
30. Of 4, 3, 2, 5, 6, and 7.
31. Of 6, 2, 7, 4, 5, and 8.

28. *CLASS II.—Problems of more than one column.*

1. What is the sum of 65, 46, and 32?

SOLUTION 1.—We write the numbers units under units and tens under tens, and begin at the right to add. 2 and 6 are 8 and 5 are 13, *units*, which equal 1 *ten* and 3 *units:* we write the 3 units under the column of units, and add the 1 ten to the column of tens. 3 and 1 are 4 and 4 are 8, and 6 are 14, *tens*, which equal 1 *hundred* and 4 *tens;* we write the 4 tens in tens' place, and the 1 hundred in hundreds' place, and we have 143.

OPERATION.

$$\begin{array}{r} 65 \\ 46 \\ \underline{32} \\ 143 \text{ Ans.} \end{array}$$

SOLUTION 2.—After the pupil is familiar with the above solution he may abbreviate it thus: 2 and 6 are 8 and 5 are 13; we write the 3 and add the 1. One and 3 are 4, and 4 are 8, and 6 are 14, which we write. He may also add by merely naming the successive sums; as, "8, 13."

EXAMPLES FOR PRACTICE.

(2.)	(3.)	(4.)	(5.)	(6.)	(7.)
43	27	37	43	58	76
38	56	25	49	36	24
81					

(8.)	(9.)	(10.)	(11.)	(12.)	(13.)
23	28	34	44	82	18
36	51	47	56	17	71
47	35	22	31	45	49

(14.)	(15.)	(16.)	(17.)	(18.)	(19.)
247	462	442	756	361	826
358	379	867	482	484	108

(20.)	(21.)	(22.)	(23.)	(24.)	(25.)
317	424	365	813	678	725
452	536	407	791	123	146
324	817	324	142	414	234

(26.)	(27.)	(28.)	(29.)	(30.)	(31.)
463	282	365	216	417	318
217	187	149	418	282	182
345	208	372	732	479	479

(32.)	(33.)	(34.)	(35.)	(36.)	(37.)
729	321	242	813	183	815
538	467	517	916	517	581
212	213	343	732	648	186
400	457	525	145	422	307

(38.)	(39.)	(40.)	(41.)	(42.)	(43.)
361	217	678	678	489	289
163	721	321	910	201	303
725	548	473	112	232	132
643	918	258	814	425	333
146	172	345	756	267	456

(44.)	(45.)	(46.)	(47.)	(48.)	(49.)
4567	1718	2526	3343	4243	1525
8910	1920	2728	5363	4546	3545
1112	2122	9303	7389	4748	5565
3456	2324	1323	4041	9505	7585

(50.)	(51.)	(52.)	(53.)	(54.)	(55.)
5960	7374	8163	8124	2185	6215
6162	5789	2738	1792	6727	8372
3646	2100	2543	8547	9858	5728
5666	4731	7342	3218	2832	6217
7869	2578	1856	4002	1479	1234

(56.)	(57.)	(58.)
48721	32173	67321
32578	82573	73214
41625	21289	84366
78321	47020	92785
47856	21832	12346

59 Find the sum of 2185+6357+4832+6719+4324.
60. Find the sum of 4344+4647+4849+5051+5253.
61. Find the sum of 6432+7253+2187+6730+5087
62. Find the sum of 2426+3275+8397+2547+8037.
63. Find the sum of 234+6721+853+8762+3739.
64. Find the sum of 834+6737+8321+123+9207.
65. Find the sum of 3246+2109+465+3712+2573

PRACTICAL PROBLEMS.

1. Mary has 15 apples and John has 23 apples; how many have they both?

SOLUTION.—If Mary has 15 apples and John has 23 apples, they both have the sum of 15 apples and 23 apples, which is 38 apples.

NOTE.—Very young pupils may say, they both have the sum of 15 apples and 23 apples, which is 38 apples.

OPERATION.

$$\begin{array}{r} 15 \\ \underline{23} \\ 38 \text{ Ans.} \end{array}$$

2. There were 25 robins on one tree and 36 robins on another tree; how many robins were there on both trees?

3. Willie has 36 cents in one pocket and 45 cents in the other; how many has he in both pockets?

4. A little boy had 37 walnuts, and then picked 56 more; how many walnuts did he then have?

5. Emma's doll cost 95 cents, and a little cradle for it cost 225 cents; how much did both cost?

6. There were 48 roses on one bush and 39 roses on another bush; how many roses were there on both bushes?

7. A little girl read 146 words one day and 178 words the next day; how many words did she read both days?

8. Harry had 246 cents in his money-box, and his uncle gave him 175 cents; how many cents had he then?

9. Peter's kite arose 436 feet, and Andrew's kite went 58 feet higher; how high did Andrew's kite arise?

10. Edward took 692 steps in going to school, and Mary took 742 steps; how many steps did they both take?

11. Mary's garden contains 47 roses, 39 pinks and 52 lilies; how many flowers are in Mary's garden?

12. Sallie spelled 25 words correctly, Jennie 36 words,

and Maggie 28 words; how many did they all spell correctly?

13. Charlie wrote 346 words last week and 378 words this week; how many words did he write in the two weeks?

14. Minnie saw 46 swallows in a flock, and Maggie saw 54 swallows in another flock; how many swallows did they both see?

15. Frank says he took 627 steps in going to school, and only 596 steps in coming from school; how many steps did he take in all?

16. My father has 6 horses, 13 cows, and 46 sheep; how many animals has he in all?

17. Emma's new reader contains 46 pictures, and Ella's contains 78 pictures; how many pictures are there in both of these readers?

18. Edward's top cost 25 cents, his whip cost 43 cents, and his ball cost 75 cents; how many cents did they all cost?

19. Albert's father owned 27 little pigs, and Peter's father owned 34 little pigs; how many little pigs had they both?

20. My father gave 215 cents for my cap, 365 cents for my vest, and 625 cents for my coat; how many cents did he give for them all?

21. One old hen had 17 little chickens, another had 15 little chickens, and another 16; how many chickens did the three hens have?

22. Henry learned seventy-five words one week and eighty-four words the next week; how many words did he learn both weeks?

23. Maria has fifty-seven cents in her money-bank, and her aunt put twenty-five cents more in the bank; how many cents did she then have?

24. There were sixteen robins in a tree, twenty-four on the barn, and thirty-nine in the meadow; how many robins were there in all?

GROUPING BY 10's, 11's, AND 9's.

Skilful accountants in adding long columns often group the figures into 10's, 11's, and 9's, and pupils should be taught to do the same. They should learn to see these groups at a glance.

1. Groups of two figures that make 10:

1	2	3	4	5	6	7	8	9
9	8	7	6	5	4	3	2	1

2. Groups of two figures that make 11:

2	3	4	5	6	7	8	9
9	8	7	6	5	4	3	2

3. Groups of two figures that make 9:

1	2	3	4	5	6	7	8
8	7	6	5	4	3	2	1

4. Add the following columns, using these groups:

4	4	1	2	2	3	8	9	5
5	6	9	7	7	7	2	2	6
3	4	2	3	5	1	7	5	2
7	5	7	8	5	8	4	6	9

5. Add the following columns, using these groups:

5	2	6	8	5	8	4	7	2
2	5	2	6	3	3	7	5	8
8	5	8	3	8	7	3	6	5
1	4	2	5	7	6	2	7	6
9	6	7	5	2	4	9	4	9

6. Groups of three figures that make 10:

1	2	1	3	2	6	4	3	1	4	2	3	1	2
2	7	3	6	6	2	3	4	4	1	3	5	8	0
7	1	6	1	2	2	3	3	5	5	5	2	1	8

7. Groups of three figures that make 11:

1	2	8	7	3	1	9	1	6	2	5	2	1	4
2	8	1	1	7	3	1	9	2	6	2	5	4	6
8	1	2	3	1	7	1	1	3	3	4	4	6	1

8. Groups of three figures that make 9:

1	1	1	2	1	3	2	2	4	1	4	2	4	4
1	7	2	6	3	5	2	3	2	4	1	4	3	0
7	1	6	1	5	1	5	4	3	4	4	3	2	5

9. Add the following columns, grouping by 10's:

2	4	4	8	1	8	5	7	8	2
8	7	5	6	2	5	7	8	6	8
1	1	2	3	8	4	1	4	4	3
3	4	0	0	4	4	6	2	5	2
6	5	8	7	6	2	3	4	1	5

10. Add the following columns, grouping by 11's:

3	6	7	6	4	7	3	7	2	8
5	5	6	8	5	6	7	5	8	7
6	2	3	2	2	3	5	4	5	6
3	0	0	2	3	3	2	4	4	4
2	9	8	7	6	5	4	3	2	1

11. Add the following, grouping by 9's:

4	5	2	2	6	4	5	5	3	5
6	8	7	5	8	7	6	8	9	6
1	2	3	3	4	3	1	2	4	2
3	0	0	2	1	1	2	1	2	6
5	7	6	4	4	5	6	6	3	1

12. Add the following, using groups:

7	5	8	2	4	1	3	2	4
3	6	4	9	9	3	9	9	6
4	2	7	5	7	5	4	6	3
6	8	5	4	2	2	6	4	8
5	5	6	7	4	7	8	5	1
5	4	2	5	5	1	3	5	9
3	6	9	6	2	2	5	3	5
7	8	4	4	7	6	5	8	1
2	3	3	3	3	4	2	2	3
8	7	7	8	6	5	7	7	6

SUBTRACTION.

29. ***Subtraction*** is the process of finding the *difference* between two numbers.

30. The ***Difference*** of two numbers is the number which, added to the smaller, equals the greater.

The *Difference* is also called the *Remainder.*

31. The ***Minuend*** is the number from which we subtract. The ***Subtrahend*** is the number to be subtracted.

32. The ***sign of Subtraction*** is —, and is read *minus.* It denotes that the number after the sign is to be subtracted from the one before it.

Case I.

33. To subtract when no figure of the subtrahend expresses more units than its figure in the minuend.

34. *CLASS I.—When the subtrahend is one figure.*

1. Subtract 4 from 9.

	SUBTRACTION.
SOLUTION.—We write the 4 under the 9 and say, 4 units from 9 units leave 5 units, which we write beneath.	9 4 5 Ans.

NOTE.—In actual practice have the pupils think 4 from 9 leaves 5, and say "4 from 9, 5," or merely "5."

EXAMPLES FOR PRACTICE.

(2.)	(3.)	(4.)	(5.)	(6.)	(7.)
5	7	5	6	8	7
2	3	3	2	5	5

(8.)	(9.)	(10.)	(11.)	(12.)	(13.)
8	7	6	9	8	9
8	4	3	6	2	7

14. Subtract 3 from 9; 6 from 17; 7 from 19; 8 from 19.

15. Subtract 2 from 14; 4 from 9; 8 from 18; 6 from 19.

16. Subtract 7 from 18; 5 from 17; 6 from 18; 5 from 16.

17. Subtract 8 from 19; 5 from 16; 9 from 19; 4 from 17.

35. *CLASS II.—When each term is two or more figures.*

1. Subtract 24 from 67.

SOLUTION.—We write the 24 under the 67, units under units, and tens under tens, and begin at the right to subtract. 4 units from 7 units leave 3 units, 2 tens from 6 tens leave 4 tens; hence the remainder is 4 tens and 3 units, or 43.

OPERATION.

67
24
43 Ans.

NOTE.—In reciting, pupils may say "4 from 7, 3;" "2 from 6, 4."

EXAMPLES FOR PRACTICE.

(2.)	(3.)	(4.)	(5.)	(6.)	(7.)
58	86	72	53	46	76
35	24	41	22	15	24

(8.)	(9.)	(10.)	(11.)	(12.)	(13.)
49	67	85	97	86	99
27	26	52	25	73	25

(14.)	(15.)	(16.)	(17.)	(18.)	(19.)
625	456	763	617	767	896
312	215	512	215	123	432

(20.)	(21.)	(22.)	(23.)	(24.)	(25.)
872	725	857	907	840	876
161	413	654	205	320	345

(26.)	(27.)	(28.)	(29.)	(30.)	(31.)
279	807	796	736	967	875
136	502	452	432	234	345

(32.)	(33.)	(34.)	(35.)	(36.)	(37.)
8763	9076	3769	5076	4872	7659
4321	4054	1546	3075	2342	3237

(38.)	(39.)	(40.)	(41.)	(42.)	(43.)
8769	4876	8275	8799	8591	5857
3257	2142	3251	2542	7230	1234

(44.)	(45.)	(46.)	(47.)	(48.)
82345	57596	72578	27397	67385
22121	21321	41362	22315	24123

(49.)	(50.)	(51.)	(52.)	(53.)
57897	67858	87578	96754	81296
21472	32721	21335	21423	20135

(54.)	(55.)	(56.)	(57.)	(58.)
253786	472589	87695	56728	98785
213123	212423	23542	21306	21342

(59.)	(60.)	(61.)	(62.)	(63.)
373967	873972	72587	95837	89976
212851	132421	51234	51321	32742

Subtract

64. 314 from 678.
65. 425 from 658.
66. 561 from 789.
67. 254 from 576.
68. 437 from 869.

Subtract

69. 1235 from 3768.
70. 3726 from 4969.
71. 2532 from 8748.
72. 4720 from 87856.
73. 12345 from 68799.

Case II.

36. To subtract when a figure in the subtrahend expresses more than the corresponding figure in the minuend.

37. *CLASS I.—When the subtrahend is one figure.*

1. Subtract 3 from 12.

OPERATION.

12
8
4 Ans.

SOLUTION.—We write the 8 under the 12; then 8 from *twelve* is *four*.

EXAMPLES FOR PRACTICE.

(2.)	(3.)	(4.)	(5.)	(6.)	(7.)	(8.)	(9.)
12	12	13	14	10	11	10	13
9	7	8	6	6	7	8	9
							4

(10.)	(11.)	(12.)	(13.)	(14.)	(15.)	(16.)	(17.)
15	15	16	13	16	17	16	14
7	8	9	7	8	9	6	5

(18.)	(19.)	(20.)	(21.)	(22.)	(23.)	(24.)	(25.)
10	17	13	11	10	19	14	16
3	6	5	4	2	9	8	7

38. *CLASS II.—When each term is two or more figures.*

1 Subtract 45 from 82.

OPERATION.

82
45
37 Ans.

SOLUTION 1.—We write the 45 under 82, placing units under units, and tens under tens, and begin at the right to subtract. We cannot subtract 5 units from 2 units; we will therefore take 1 ten from the 8 tens, leaving 7 tens; 1 ten equals 10 units, which added to 2 units equals 12 units; 5 units from 12 units leave 7 units; 4 tens from 7 tens leave 3 tens; hence, the remainder is 37.

SOLUTION 2.—We cannot take 5 units from 2 units; we will therefore add 10 units to the 2 units, making 12 units; 5 units from 12

units leave 7 units. Now, since we have added 10 units, or 1 ten, to the minuend, our remainder will be 1 ten too large; hence, we must add 1 ten to the subtrahend; 1 ten and 4 tens are 5 tens, 5 tens from 8 tens leave 3 tens.

NOTE.—In practice we solve thus; 5 from 2 we cannot take, but 5 from 12 leaves 7, 4 and 1 are 5, and 5 from 8 leaves 3.

39. From the preceding explanations we have the following general rule.

RULE.—1. *Write the subtrahend under the minuend, placing terms of the same order in the same column, and draw a line beneath.*

2. *Begin at the right and subtract each term of the subtrahend from the corresponding term of the minuend, writing the remainder beneath.*

3. *If any term of the subtrahend is greater than the corresponding term of the minuend, add* 10 *to the latter, and then subtract. Add* 1 *to the next term of the subtrahend, and proceed as before.*

40. PROOF.—Add the remainder to the subtrahend; the sum will equal the minuend if the work is correct.

EXAMPLES FOR PRACTICE.

(2.)	(3.)	(4.)	(5.)	(6.)	(7.)
73	64	32	41	53	62
25	27	14	26	28	28
(8.)	(9.)	(10.)	(11.)	(12.)	(13.)
75	31	57	63	87	95
26	18	29	45	28	59
(14.)	(15.)	(16.)	(17.)	(18.)	(19.)
87	75	63	77	87	94
39	38	25	48	59	49
(20.)	(21.)	(22.)	(23.)	(24.)	(25.)
72	84	70	81	90	97
27	48	17	18	39	79

(26.)	(27.)	(28.)	(29.)	(30.)	(31.)
342	573	692	545	826	357
124	245	457	328	252	183
(32.)	(33.)	(34.)	(35.)	(36.)	(37.)
573	748	835	968	839	538
248	375	573	675	584	394
(38.)	(39.)	(40.)	(41.)	(42.)	(43.)
659	839	547	658	735	848
475	583	284	372	373	539
(44.)	(45.)	(46.)	(47.)	(48.)	(49.)
524	752	845	307	456	450
356	387	579	138	387	382
(50.)	(51.)	(52.)	(53.)	(54.)	(55.)
854	943	607	500	704	403
396	765	309	325	507	285
(56.)	(57.)	(58.)	(59.)	(60.)	(61.)
726	857	735	792	807	650
387	389	558	295	328	357
(62.)	(63.)	(64.)	(65.)	(66.)	(67.)
3876	6385	6735	4076	4070	4135
2379	3527	2547	3128	2137	1216
(68.)	(69.)	(70.)	(71.)	(72.)	(73.)
8672	5283	8175	2534	6735	7219
3728	2426	2836	1235	5376	1972
(74.)	(75.)	(76.)	(77.)	(78.)	(79.)
8522	7135	6347	8135	7345	4372
6243	1872	2563	2453	2876	2583
(80.)	(81.)	(82.)	(83.)	(84.)	(85.)
35672	43768	87253	73875	63527	53413
23828	24235	34365	38376	14238	28401

(86.)	(87.)	(88.)	(89.)	(90.)	(91.)
73285	20307	87004	76500	40500	37201
43836	15231	34523	43654	37254	23534

(92.)	(93.)	(94.)	(95.)	(96.)	(97.)
83030	90304	50310	60204	70000	100000
76543	40372	30311	30205	32463	1

PRACTICAL PROBLEMS.

1. Mary had 25 roses and gave Sarah 12 of them; how many did Mary then have?

SOLUTION.—If Mary had 25 roses and gave Sarah 12 of them, Mary then had the difference between 25 roses and 12 roses, which is 13 roses.

NOTE.—Quite young pupils may say, Mary then had the difference between 25 roses and 12 roses, which is 13 roses.

OPERATION.

25
12
13 Ans.

2. Willie had 34 cents and gave James 18 cents; how many cents did Willie then have?

3. A little girl had 54 pins and gave her cousin 27 of them; how many did she have remaining?

4. Fifty little robins were sitting on a tree, and 23 of them flew away; how many were then left?

5. There were 96 peaches on an old peach-tree, and a gust of wind blew 37 of them off; how many then remained on the tree?

6. Henry's top and ball cost 120 cents; how much did the top cost, if the ball cost 75 cents?

7. Emma's doll and its little cradle cost 320 cents, and the doll cost 95 cents; how much did the cradle cost?

8. There were 87 roses on two rose-bushes; how many roses were there on the second bush, if there were 39 roses on the first bush?

9. A little girl read 324 words in two days; if she read 146 words one day, how many did she read the other day?

10. Edward and Mary together took 1434 steps in going to school; how many steps did Mary take, if Edward took 692 steps?

11. Minnie had 372 cents in her money-bank, and took out 164 cents to give to a little beggar-girl; how many cents remained?

12. Andrew's kite arose 494 feet, and this was 58 feet higher than Peter's kite went; how high did Peter's kite fly?

13. Charlie wrote 724 words in two weeks; he wrote 346 words the first week; how many words did he write the second week?

14. Mary's new reader contains 76 pictures, and Fanny's contains 92 pictures; how many does Fanny's contain more than Mary's?

15. Two little girls picked 74 quarts of blackberries one summer; if one picked 37 quarts, how many quarts did the other pick?

16. Thomas said he counted 283 crows in his father's cornfield; he threw a stone and scared 126 away; how many then remained?

17. Floy and Eugie together took 3000 steps one day; if Floy took 1786 steps, how many steps did Eugie take?

18. Effie and Eddie counted their chestnuts and found they together had 1232; now, if Eddie had 675, how many had Effie?

19. Mary's mother bought her an arithmetic and slate for 125 cents; if the slate cost 45 cents, what did the arithmetic cost?

20. Herbert's father bought him a cap and coat for 850 cents; he paid 125 cents for the cap; how much did he pay for the coat?

21. Mr. Nelson's horse and carriage cost four hundred dollars; what did the horse cost, if the carriage cost two hundred and twenty-five dollars?

22. Two little boys picked eighty-four quarts of blackberries one summer; if one picked forty-seven quarts, how many quarts did the other pick?

23. Mr. Barton raised two thousand bushels of wheat and rye; how much rye did he raise, if he raised five hundred and sixty-five bushels of wheat?

PRACTICAL PROBLEMS.

1. A man had 78 cows and sold 24 of them; how many cows remained?

SOLUTION.—If a man had 78 cows and sold 24, there remained the difference between 78 and 24, which we find by subtracting is 54.

NOTE.—Quite young pupils may merely say, there remains the difference between 78 and 24, which is 54.

OPERATION

$$\begin{array}{r} 78 \\ \underline{24} \\ 54 \end{array}$$ Ans.

2. A boy had 150 cents and spent 75 cents; how many cents then remained?

3. A man had 325 apples and sold 180 apples; how many had he then?

4. Henry had 1735 dollars, lent his brother 854 dollars; how many dollars remained?

5. A bought 570 horses and sold 295 of them; how many remained unsold?

6. A and B together had 7256 acres of land; how many had B if A had 3627?

7. Two men have 8570 bushels of grain, and the first has 2846 bushels; how many has the second?

8. Washington was born 1732 and died 1799; how old was he at his death?

9. John Adams was born 1735 and died 1826; how old was he at his death?

10 Jefferson was born 1743 and died 1826; how old was he at his death?

11. Madison was born 1758 and died 1836; how old was he at his death?

12. Monroe was born 1758 and died 1831; how old was he at his death?

13. John Quincy Adams was born 1767 and died 1848; how old was he at his death?

14. Jackson was born 1767 and died 1845; how old was he at his death?

15. A has 5480 bushels of oats, which is 975 bushels more than B has; how many bushels has B?

16. In an army of 50000 men 628 were killed and 2596 wounded; how many remained unhurt?

17. A farmer had 234 hens and bought 367, and then sold 489; how many then remained.

18. A merchant sold goods to the amount of 7580 dollars and gained 1396 dollars; what did the goods cost?

19. A and B have each 1840 acres of land; A sells B 895 acres; how many has each then?

20. A farmer has 1346 sheep and 849 lambs; how many more sheep has he than lambs?

21. Mary and Eliza have each 789 cents; if Eliza gives Mary 247 cents, how many will each then have?

22. Subtract six hundred and seventy-eight from nine hundred and four.

23. Add seven hundred and fifteen to five hundred and seventy-three, and subtract the sum from two thousand.

24. Find the sum of one thousand and ninety-six and five hundred and forty-five, and subtract it from three thousand.

25. Frank solved four hundred and sixteen problems, and Fanny solved five hundred and three problems: how many did Fanny solve more than Frank?

26. A farmer had 2346 bushels of wheat; he sold one man 687 bushels and another man 1560 bushels; how many bushels did he sell? how many remained?

PRACTICAL PROBLEMS in Addition and Subtraction.

1. If I have 75 cents in my money-bank, and my uncle puts in 26 cents, how much will be in it then?

2. If Willie reads 125 words this week and 187 words next week, how many words will he read in all?

3. My father had 236 little chickens, and a mink killed 48 of them; how many remained?

4. If I have 438 dollars and give my sister 246 dollars, how much will I have remaining?

5. Mary's father had 360 acres of land and sold 125 acres; how many acres did he then have?

6. Peter had 467 dollars and lent his brother 185 dollars; how much did he then have?

7. I have 365 cents in my money-bank; how many must I put in that there may be 400 cents in it?

8. Sallie had 72 cents and her brother gave her enough to make her money 134 cents; how much did her brother give her?

9. Carrie's brother teased her because she couldn't tell how many she must add to 245 to make 400; can you tell?

10. Matilda had 120 cents, her mother gave her 236 cents, and then she lent her brother 248 cents; how many cents did she then have?

11. Fannie picked 236 chestnuts, her little brother gave her 78 chestnuts, and she gave 95 to her schoolmates; how many chestnuts remained?

12. Mary cried because she couldn't tell her teacher how many she must add to 367 to make 500; tell me how many it is.

13. One morning in going to school I took 726 steps; how many more would I have taken if I had taken 1000 in all?

14. My kite was up in the air 436 feet, it then fell 185 feet, and then arose 260 feet; how high was it then?

BUSINESS PROBLEMS.

1. I went to a store and bought a book for 87 cents and a slate for 35 cents; what did I pay for both of them?

2. My mother took me to a store and bought me a top for 15 cents, a cap for 75 cents, and a knife for 45 cents; what did they all cost?

3. William's slate cost 26 cents, his arithmetic 55 cents, his reading-book 48 cents, and his spelling-book 37 cents; what did they all cost?

4. I went to a store and bought a knife for 56 cents and gave the storekeeper a dollar bill (100 cents) to pay for it; how much change did he give me back?

5. Mary bought a flower-vase for 375 cents, and handed the storekeeper a five-dollar bill (500 cents) to pay for it; how much change should she have received?

6. Mr. Barnes paid 75 dollars for his watch, and sold it so that he gained 12 dollars; what did he receive for it?

7. Martha's new shawl cost 875 cents; if she should sell it so as to gain 125 cents, what would she receive for it?

8. Mr. Taylor's new house cost him 3675 dollars, and he sold it for 565 dollars more than it cost him; what did he receive for it?

9. My father bought a cow for 38 dollars and sold her for 52 dollars; how much did he gain on the cow?

10. Robert Stewart had a coat which cost him 45 dollars; he sold it to Edward Taylor for 37 dollars; how much did he lose?

11. Harry Hartman sold his watch for 67 dollars and lost by the sale 15 dollars; what did the watch cost him?

12. Mary's papa gave her a 5 dollar bill to go a shopping; she bought a fan for 75 cents, some silk for 165 cents, and a pair of gloves for 125 cents; how much change did she bring home?

PRACTICAL PROBLEMS
in Addition and Subtraction.

1. What is the value of $675+432+285+672$?

2. What is the value of $362+486+721-367$?

3 What is the value of $473+325+604-1206$?

4. What is the value of $3072+4861+2075-6785$?

5. Subtract 1678 from the sum of 985 and 863.

6. Subtract the sum of 265 and 381 from the sum of 281 and 678.

7. Subtract $218+318+418$ from $379+279+479$.

8. A having 475 dollars earned 220 dollars and then spent 567; how much remained?

9. Newspapers were first published in 1630; how long have they been published?

10. Quills were first used for writing about the year 636; how long is it since?

11. Cotton was first planted in the United States about the year 1769; how many years since?

12. Glass windows, it is said, were first used in England in 1180; how long is it since then?

13. A sold his farm for 12450 dollars, which was 1680 dollars more than it cost; how much did it cost?

14. A gave 6500 dollars for his farm and 2560 dollars for his house, and sold them for 12000; what was the gain?

15. A farmer had 5600 bushels of corn, and sold 1850 bushels to A and 2810 to B; how much remained?

16. The area of Maine is 30000 square miles, and of New York 46000; how much larger is New York than Maine?

17. The area of Massachusetts is 7800 square miles, and of Pennsylvania 46000 square miles; how much larger is Pennsylvania than Massachusetts?

18. How much larger are Pennsylvania and Maine together than New York and Massachusetts together?

INTRODUCTION TO MULTIPLICATION AND DIVISION.

If pupils are not already familiar with the *elementary products* and *quotients*, they should be drilled on exercises similar to the following.

Lead the pupil to see the relation between multiplication and division, so that when he *knows a product* he can immediately *derive a quotient.*

1. Since 3 and 3 are 6, how many times 3 make 6? How many are 2 times 3?

2. How many are 2 times 2? 2 times 3? 2 times 4? 2 times 5? 2 times 6? 2 times 7? 2 times 8? 2 times 9? 2 times 10? 2 times 11? 2 times 12?

3. Since two times 3 make 6, how many 3's are there in 6? How often is 3 contained in 6?

4. How many times is 2 contained in 4? 4 in 8? 5 in 10? 6 in 12? 7 in 14? 8 in 16? 9 in 18? 10 in 20? 11 in 22? 12 in 24?

5. How many are 3 times 1? 3 times 2? 3 times 3? 3 times 4? 3 times 5? 3 times 6? 3 times 7? 3 times 8? 3 times 9? 3 times 10? 3 times 11? 3 times 12?

6. How many times is 1 contained in 3? 2 in 6? 3 in 9? 4 in 12? 5 in 15? 6 in 18? 7 in 21? 8 in 24? 9 in 27? 10 in 30? 11 in 33? 12 in 36?

7. Make the table of "two times," and derive the table of quotients from it, as below. Also the table of "three times."

$2\times1=2$,	$2\div2=1$,	$3\times1=3$,	$3\div3=1$,
$2\times2=4$,	$4\div2=2$,	$3\times2=6$,	$6\div3=2$,
$2\times3=6$,	$6\div2=3$,	$3\times3=9$,	$9\div3=3$,
$2\times4=8$,	$8\div2=4$,	$3\times4=12$,	$12\div3=4$,
etc.	etc.	etc.	etc.

NOTE.—The pupils will make the multiplication table by addition; thus, to find 3 times 4, find the sum of $4+4+4=12$. Subsequently pupils should be led to see that each following product is obtained by adding the number multiplied to the previous product; thus, since 6 times 4 is 24, 6 times 5 is 6 more than 24, or 30.

8. Fill out the following to 4×12 and 5×12:

$4\times1=4$,	$4\div4=1$,	$5\times1=5$,	$5\div5=1$,
$4\times2=8$,	$8\div4=2$,	$5\times2=10$,	$10\div5=2$,
$4\times3=12$,	$12\div4=3$,	$5\times3=15$,	$15\div5=3$,
$4\times4=16$,	$16\div4=4$,	$5\times4=20$,	$20\div5=4$,
etc.	etc.	etc.	etc.

9. Fill out the following to 6×12 and 7×12:

$6\times 1=$	$6\div 6=$	$7\times 1=$	$7\div 7=$	$8\times 1=$	$8\div 8=$
$6\times 2=$	$12\div 6=$	$7\times 2=$	$14\div 7=$	$8\times 2=$	$16\div 8=$
$6\times 3=$	$18\div 6=$	$7\times 3=$	$21\div 7=$	$8\times 3=$	$24\div 8=$
etc.	etc.	etc.	etc.	etc.	etc.

10. Fill out the following to 9×12, 10×12, and 11×12:

$9\times 1=$	$9\div 9=$	$10\times 1=$	$10\div 10=$	$11\times 1=$	$11\div 11=$
$9\times 2=$	$18\div 9=$	$10\times 2=$	$20\div 10=$	$11\times 2=$	$22\div 11=$
$9\times 3=$	$27\div 9=$	$10\times 3=$	$30\div 10=$	$11\times 3=$	$33\div 11=$
etc.	etc.	etc.	etc.	etc.	etc.

11. Make in a similar manner the table of 12 times, with the derived quotients.

12. Recite orally the table of 2 times and its derived quotients. Do the same with 3 times, 4 times, 5 times, etc.

13. Find the results of the following:

$8\times 3+6=$	$4\times 5+3=$	$4\times 9-6=$	$6\times 2-4=$
$6\times 5+4=$	$6\times 4+5=$	$7\times 4-5=$	$6\times 7-8=$
$7\times 3+8=$	$3\times 8+7=$	$6\times 6-6=$	$5\times 9-6=$

14. Find the results of the following:

$8\div 2+4=$	$15\div 3+6=$	$21\div 3+7=$	$35\div 7-2=$
$9\div 3+5=$	$18\div 6+4=$	$24\div 6+8=$	$40\div 5-4=$
$10\div 2+6=$	$20\div 5+6=$	$30\div 6-2=$	$48\div 8-6=$

15. If 1 apple costs 4 cents, what will 2 apples cost?

SOLUTION.—If 1 apple costs 4 cents, 2 apples will cost 2 times 4 cents, or 8 cents.

16. If 1 pencil costs 6 cents, what will 4 pencils cost?

17. What will 5 chairs cost at 4 dollars apiece?

18. What will 6 melons cost at 8 cents apiece?

19. What will 8 yards of carpet cost at 3 dollars a yard?

20. If 1 apple costs 2 cents, how many apples can you buy for 8 cents?

SOLUTION.—If 1 apple costs 2 cents, for 8 cents I can buy as many apples as 2 is contained times in 8, which are 4.

21. If 1 hat costs 3 dollars, how many hats can you buy for 12 dollars?

22. If 1 chair costs 4 dollars, how many chairs can you buy for 20 dollars?

23. How many melons can you buy for 48 cents, at 8 cents apiece?

MULTIPLICATION.

41. ***Multiplication*** is the process of finding the *product* of two numbers.

42. The ***Product*** of two numbers is the result obtained by taking one number as many times as there are units in another.

43. The ***Multiplicand*** is the number to be multiplied. The ***Multiplier*** is the number by which we multiply.

44. ***The sign of Multiplication*** is $\times$, and is read *multiplied by;* thus, $4 \times 3 = 12$ means 4 *multiplied by* 3 *equals* 12. The 4 is the multiplicand. 3 is the multiplier, and 12 is the product.

NOTE TO TEACHERS.—If the pupils are not familiar with the Multiplication Table, let them be drilled upon it.

CASE I.

45. When the multiplier is one figure.

CLASS I.—When no product exceeds nine.

1. Multiply 34 by 2.

SOLUTION.—We write the multiplier under the multiplicand, and begin at the right to multiply. 2 times 4 units are 8 units; we write the 8 units in units' place. 2 times 3 tens are 6 tens; we write the 6 tens in tens' place.

OPERATION.

$$\begin{array}{r} 34 \\ \underline{2} \\ 68 \text{ Ans.} \end{array}$$

(2.)	(3.)	(4.)	(5.)
32	24	14	41
2	2	2	2

(6.)	(7.)	(8.)	(9.)
21	12	23	32
3	3	3	8

46. *CLASS II.—When some of the products exceed nine.*

1. Multiply 56 by 4.

OPERATION.
56
4
224 Ans.

SOLUTION 1.—We write the multiplier under the multiplicand and begin at the right to multiply. 4 times 6 units are 24 units, which equal 4 units and 2 tens. We write the 4 units in units place, and reserve the 2 tens to add to the next product. 4 times 5 tens are 20 tens, plus the 2 tens equal 22 tens, which equal 2 tens and 2 hundreds, which we write in their proper places. Hence the product is 224.

SOLUTION 2.—4 times 6 are 24; we write the 4 and add the 2 to the next product. 4 times 5 are 20, and 2 added equal 22; hence the product is 224. From this we have the following

RULE.—1. *Write the multiplier under the multiplicand, and draw a line beneath.*

2. *Begin at the right, and multiply each term of the multiplicand by the multiplier, carrying as in addition.*

(2.)	(3.)	(4.)	(5.)	(6.)
25	36	47	73	28
3	2	3	2	4
(7.)	(8.)	(9.)	(10.)	(11.)
63	75	36	27	43
5	4	5	6	5
(12.)	(13.)	(14.)	(15.)	(16.)
75	86	92	76	84
2	3	4	5	6
(17.)	(18.)	(19.)	(20.)	(21.)
73	47	76	85	73
5	6	7	8	8
(22.)	(23.)	(24.)	(25.)	(26.)
234	425	673	723	351
3	4	5	6	7
(27.)	(28.)	(29.)	(30.)	(31.)
425	314	421	630	854
6	7	8	7	8

(32.)	(33.)	(34.)	(35.)	(36.)
256	375	873	358	725
4	6	7	8	7

(37.)	(38.)	(39.)	(40.)	(41.)
581	809	394	908	765
7	8	6	9	8

Multiply	Multiply
42. 3124 by 4.	50. 13257 by 2.
43. 2856 by 5.	51. 36072 by 3.
44. 7863 by 6.	52. 85761 by 4.
45. 2185 by 7.	53. 35167 by 5.
46. 4182 by 8.	54. 84307 by 6.
47. 3075 by 8.	55. 30754 by 7.
48. 4107 by 9.	56. 21836 by 8.
49. 7685 by 6.	57. 35168 by 9.

Case II.

47. When the multiplier consists of two or more figures.

48. *CLASS I.—When the multiplier consists of two figures.*

1. Multiply 64 by 23.

OPERATION.
64
23
192
128
1472 Ans.

Solution 1.—We write the multiplier under the multiplicand, placing units under units, and tens under tens, and begin at the right to multiply. 3 times 4 units are 12 units, which equals 1 ten and 2 units; we write the units under the 3, and reserve the 1 ten to add to the next product. 3 times 6 tens are 18 tens, and 1 ten added equals 19 tens, or 1 hundred and 9 tens, which we write in their proper places. Multiplying 64 by 2 in the same manner, we have 128, and since the 2 is 2 *tens* we have 128 tens, which we write in its proper place; then, adding the two products, we have 1472.

Solution 2.—Three times 4 are 12; we write the 2 and carry the

1: three times 6 are 18, plus the 1 equals 19; which we write. Then, 2 times 4 are 8, which we write under the 2, and 2 times 6 are 12, which we write beside the 8.

RULE.—1. *Write the multiplier under the multiplicand, placing terms of the same order in the same column, and draw a line beneath.*

2. *Begin at the right, and multiply the multiplicand by each term of the multiplier, writing the first term of each product under the term of the multiplier which produces it.*

3. *Add the partial products, and their sum will be the entire product.*

PROOF.—Multiply the multiplier by the multiplicand; if the two results agree, the work is probably correct

(2.)	(3.)	(4.)	(5.)	(6.)	(7.)
38	43	73	81	29	57
23	24	35	67	82	75
(8.)	(9.)	(10.)	(11.)	(12.)	(13.)
87	39	87	29	123	245
28	43	52	92	37	32
(14.)	(15.)	(16.)	(17.)	(18.)	(19.)
436	534	427	426	534	672
43	43	35	43	45	46
(20.)	(21.)	(22.)	(23.)	(24.)	(25.)
725	634	807	475	709	493
42	47	37	54	38	82
(26.)	(27.)	(28.)	(29.)	(30.)	(31.)
756	762	675	467	762	812
93	48	39	37	62	45

(32.)	(33.)	(34.)	(35.)	(36.)	(37.)
1234	2341	6724	6357	7138	2536
28	35	42	35	52	25

(38.)	(39.)	(40.)	(41.)	(42.)	(43.)
6347	8192	4736	4825	3121	4073
46	73	63	72	37	46

Multiply

44. 6538 by 83.
45. 7384 by 45.
46. 2185 by 67.
47. 3407 by 82.
48. 3584 by 46.

Multiply

49. 4175 by 28.
50. 7186 by 85.
51. 8391 by 94.
52. 2187 by 89.
53. 6543 by 98.

49. *CLASS II.—When the multiplier consist of three figures.*

(1.)	(2.)	(3.)	(4.)	(5.)
4126	5731	7351	1375	5379
234	243	432	342	423

(6.)	(7.)	(8.)	(9.)	(10.)
6725	2183	7321	8193	2147
345	544	265	475	813

(11.)	(12.)	(13.)	(14.)	(15.)
2143	8192	2435	4167	8246
227	426	146	245	642

(16.)	(17.)	(18.)	(19.)	(20.)
7346	7516	8927	1928	2076
643	571	352	816	437

(21.)	(22.)	(23.)	(24.)	(25.)
4752	7385	8492	2937	6473
185	218	537	439	567

Multiply

26. 14651 by 283.
27. 31251 by 625.
28. 36782 by 224.
29. 43678 by 452.
30. 36507 by 634.
31. 40725 by 365.
32. 32107 by 681.
33. 25697 by 329.
34. 42046 by 456.

Multiply

35. 28352 by 345.
36. 41678 by 287
37. 34073 by 435.
38. 40735 by 628.
39. 29304 by 789.
40. 90705 by 897.
41. 43445 by 678.
42. 37436 by 835.
43. 88888 by 789.

50. *CLASS III.—When the multiplier consists of more than three figures.*

(1.)	(2.)	(3.)	(4.)	(5.)	(6.)
4137	3642	6724	4183	3645	4526
2185	2531	3625	2426	2841	2182

(7.)	(8.)	(9.)	(10.)	(11.)	(12.)
3482	2846	3707	4172	2882	8567
2534	2528	2851	2174	2773	3178

(13.)	(14.)	(15.)	(16.)	(17.)	(18.)
5185	9187	4785	8197	4376	8765
8763	2567	7372	1846	5273	5678

Multiply

19. 28751 by 3146.
20. 17346 by 2435.
21. 21307 by 3147.
22. 85276 by 3452.

Multiply

23. 72509 by 3167.
24. 85216 by 2431.
25. 73519 by 4735.
26. 81897 by 3456.

Multiply	Multiply
27. 21346 by 31452.	31. 10786 by 31672.
28. 47309 by 45233.	32. 47396 by 73462.
29. 25737 by 63252.	33. 76448 by 54173.
30. 43629 by 28516.	34. 28354 by 31867.

51. *CLASS IV.—When one or both terms contain ciphers.*

1. Multiply 5721 by 3006; also, 37000 by 2400.

OPERATION.	OPERATION.
5721	37000
3006	2400
34326	148
17163	74
17197326	88800000

NOTE.—In the first example, pass over the naughts, placing the right-hand figure of the product by 3 directly under the 3. In the second problem, we multiply by the significant figures, and then annex the naughts to the product.

Multiply	Multiply
2. 3678 by 204.	11. 4500 by 2800.
3. 4107 by 307.	12. 67000 by 450.
4. 4178 by 1005.	13. 96000 by 2800.
5. 8675 by 3007.	14. 87000 by 4800.
6. 7276 by 6008.	15. 73500 by 32000.
7. 4136 by 2305.	16. 86700 by 47200.
8. 8449 by 3046.	17. 32800 by 346000.
9. 4592 by 5607.	18. 70900 by 407100.
10 8124 by 4801.	19. 85900 by 1030600.

20. Multiply four thousand six hundred and ten by seven thousand and forty.

EXAMPLES IN MULTIPLICATION.

1. If one orange cost 8 cents, what will 7 oranges cost at the same rate?

SOLUTION.—If one orange cost 8 cents, 7 oranges will cost 7 times 8 cents, which are 56 cents.

OPERATION.

$$\begin{array}{r} 8 \\ \underline{7} \\ 56 \text{ Ans.} \end{array}$$

2. If one pig cost 7 dollars, what will 6 pigs cost at the same rate?

3. If a yard of muslin cost 37 cents, what will 8 yards cost at the same rate?

4. If a boy writes 36 words in a day, how many will he write in 13 days?

5. What must I pay for 15 cows, if I pay 28 dollars for each cow?

6. If Henry takes 42 steps in a minute, how many steps will he take in 15 minutes?

7. If a car runs 25 miles in an hour, how far will it run in 12 hours?

8. If a boy learns 14 new words each day, how many will he learn in 11 days?

9. Mary has 14 rose-bushes in her garden, and on each bush there are 26 roses; how many roses on all?

10. How much must I pay for 16 pounds of tea, at the rate of 78 cents a pound?

11. What are nine loads of hay worth, at the rate of 23 dollars a load?

12. If one cord of wood is worth six dollars, how much are 18 cords of wood worth?

13. How many marbles will 7 boys have, if each boy has 12 marbles?

14. At the rate of 45 miles a day, how far will a person travel in 23 days?

15 If Henry can count 65 in a minute, how many can he count in 26 minutes?

PRACTICAL EXAMPLES
in Multiplication.

1. What cost 24 horses at 245 dollars each?

SOLUTION.—If one horse cost 245 dollars, 24 horses cost 24 times 245 dollars, which by multiplying we find to be 5880 dollars.

2. If a boat sails 246 miles in one day, how far will it sail in 26 days?

3. If in one book there are 364 pages, how many pages in 18 such books?

4. In one barrel of flour there are 196 pounds; how many pounds in 25 barrels of flour?

5. How much will 42 horses cost, at the rate of 150 dollars apiece?

6. If an acre of land is worth 218 dollars, how much will 76 acres cost?

7. If in an orchard there are 32 rows of trees with 46 trees in a row, how many trees in all?

8. A man bought 326 horses and 36 times as many sheep; how many sheep did he buy?

9. What cost 125 yards of cloth at the rate of 325 cents a yard?

10. How much will 236 bushels of wheat cost, at 175 cents a bushel?

11. There are 1760 yards in one mile; how many yards in 12 miles?

12. There are 5280 feet in a mile; how many feet in 18 miles?

13. There are 660 feet in one furlong; how many feet in 26 furlongs?

14. There are 5760 grains in one pound Troy; how many grains in 137 pounds?

15. There are 256 drams in an ounce; how many drams in 420 ounces?

16. There are 198 inches in one rod; how many inches in 76 rods?

PRACTICAL PROBLEMS
in Multiplication.

1. How much cost 75 barrels of flour, at 7 dollars a barrel?

	OPERATION.
SOLUTION.—If 1 barrel cost 7 dollars, 75 barrels will cost 75 times 7 dollars, which are 525 dollars.	75 7 525 Ans.

NOTE.—In practice, we multiply the 75 by 7, since it is more convenient to use the smaller number as the multiplier.

2. How much will 436 bushels of potatoes cost, at 48 cents a bushel?

3. How much will 847 bushels of corn cost, at 56 cents a bushel?

4. How much will 936 yards of muslin cost, at 37 cents a yard?

5. A drover bought 4896 pigs, at 9 dollars each; what did they cost?

6. How much will 3686 grammars cost, at 54 cents apiece?

7. At 6 cents a quart, what will 3678 quarts of milk cost? 9876 quarts?

8. There are 1728 pins in a great gross; how many pins in 256 great gross?

9. There are 231 cubic inches in a wine gallon; how many inches in 48 wine gallons?

10. There are 282 cubic inches in a beer gallon; how many cubic inches in 345 beer gallons?

11. There are 4840 square yards in one acre; how many square yards in 365 acres?

12. There are 5280 feet in a mile; how many feet in 156 miles?

13. There are 63360 inches in a mile; how many inches in 640 miles?

14. There are 5280 feet in a mile; how many feet in the diameter of the earth, if it is 7912 miles?

DIVISION.

52. ***Division*** is the process of finding the *quotient* of two numbers.

53. The ***Quotient*** of two numbers is the number which shows how many times one number contains the other.

54. The ***Dividend*** is the number which contains the other.

55. The ***Divisor*** is the number contained in the dividend.

56. ***The sign of Division*** is ÷, and is read *divided by.* It shows that the number on the left is to be divided by the one on the right.

57. There are two methods of performing division, called ***Short Division*** and ***Long Division.***

NOTE TO TEACHERS.—If the pupils are not familiar with the elementary quotients, the teacher will drill them on these quotients as derived from the multiplication table.

SHORT DIVISION.

58. ***Short Division*** is the method of dividing when the partial dividends are not written.

CASE I.

59. When the divisor is one figure and the result one of the elementary quotients.

1. How many times is 2 contained in 6?

OPERATION.

$$\begin{array}{r} 2\underline{)6} \\ 3 \end{array}$$

SOLUTION 1.—We write the 6, draw a line beneath and a curve to the left, and place the 2 to the left of the curve. *Two* is contained in 6, *three* times, since 3 times 2 are 6. We write the quotient 3 beneath the dividend.

SOLUTION 2.—Two is contained in 6 three times, with no remainder.

(2.) 2)8	(3.) 3)6	(4.) 2)4	(5.) 3)9	(6.) 4)8	(7.) 2)10
(8.) 2)12	(9.) 3)12	(10.) 2)14	(11.) 3)18	(12.) 2)20	(13.) 2)22
(14.) 2)24	(15.) 3)21	(16.) 3)27	(17.) 3)30	(18.) 3)36	(19.) 3)33
(20.) 4)16	(21.) 4)24	(22.) 4)28	(23.) 4)20	(24.) 4)36	(25.) 4)48
(26.) 5)15	(27.) 5)25	(28.) 5)35	(29.) 5)20	(30.) 5)40	(31.) 5)55
(32.) 5)60	(33.) 6)12	(34.) 6)24	(35.) 6)36	(36.) 6)48	(37.) 6)60
(38.) 7)21	(39.) 7)35	(40.) 7)49	(41.) 7)63	(42.) 7)77	(43.) 7)84
(44.) 8)16	(45.) 8)64	(46.) 8)56	(47.) 8)40	(48.) 8)72	(49.) 8)96
(50.) 9)27	(51.) 9)45	(52.) 9)63	(53.) 9)81	(54.) 9)108	(55.) 9)99
(56.) 10)40	(57.) 10)80	(58.) 11)55	(59.) 11)77	(60.) 12)60	(61.) 12)84

Note.—We include 10, 11, and 12 in the case, as they are included in the table of elementary products and quotients.

Case II.

60. When the divisor is one figure and there are no remainders.

1. Divide 46 by 2.

SOLUTION 1.—2 is contained in 4 tens 2 tens times. 2 is contained in 6 units 3 units times; hence the quotient is 23.

OPERATION.
$$\begin{array}{r} 2\overline{)46} \\ \hline 23 \end{array}$$

SOLUTION 2.—2 is contained in 4, 2 times; 2 is contained in 6, 3 times.

(2.)	(3.)	(4.)	(5.)	(6.)	(7.)
2)42	2)48	2)26	2)64	2)84	2)86
(8.)	**(9.)**	**(10.)**	**(11.)**	**(12.)**	**(13.)**
2)82	3)36	3)69	3)96	3)90	3)39
(14.)	**(15.)**	**(16.)**	**(17.)**	**(18.)**	**(19.)**
4)48	4)44	4)88	4)40	4)80	5)50

(20.)	(21.)	(22.)	(23.)	(24.)
2)428	2)228	2)848	2)408	3)369
(25.)	**(26.)**	**(27.)**	**(28.)**	**(29.)**
3)693	3)906	3)609	3)930	4)480
(30.)	**(31.)**	**(32.)**	**(33.)**	**(34.)**
4)804	4)408	3)669	4)488	2)880
(35.)	**(36.)**	**(37.)**	**(38.)**	**(39.)**
2)804	3)936	2)886	2)468	3)603

Case III.

61. When the divisor is one figure and there are remainders.

1. Divide 7 by 3.

SOLUTION 1.—Three is contained in 7, 2 times, which we write under the 7; and since 2 times 3 are 6, and 7 is 1 more than 6, hence 3 is contained in 7, 2 times, and 1 remaining, which we write after the 2 with the sign + before it.

OPERATION.

$$\begin{array}{l} 3\underline{)7} \\ \;\;2+1 \end{array}$$

SOLUTION 2.—3 is contained in 7, 2 times. 2 times 3 are 6, 6 from 7 leaves 1; hence the quotient is 2, with a remainder of 1.

(2.) 2)9	(3.) 2)11	(4.) 2)19	(5.) 2)21	(6.) 2)13	(7.) 2)25
(8.) 3)5	(9.) 3)11	(10.) 3)8	(11.) 3)14	(12.) 3)17	(13.) 3)23
(14.) 4)11 2	(15.) 4)17	(16.) 4)22	(17.) 4)37	(18.) 4)43	(19.) 4)27
(20.) 5)12	(21.) 5)19	(22.) 5)28	(23.) 5)38	(24.) 5)47	(25.) 5)58
(26.) 6)15	(27.) 6)21	(28.) 6)35	(29.) 6)51	(30.) 6)65	(31.) 6)71
(32.) 7)23	(33.) 7)29	(34.) 7)38	(35.) 7)46	(36.) 7)58	(37.) 7)80
(38.) 8)23	(39.) 8)19	(40.) 8)28	(41.) 8)36	(42.) 8)47	(43.) 8)93
(44.) 9)31	(45.) 9)26	(46.) 9)52	(47.) 9)61	(48.) 9)70	(49.) 9)83

CASE IV.

62. When the quotient contains several figures and there are successive remainders.

1. Divide 536 by 2.

SOLUTION 1.—2 is contained in 5 hundreds 2 hundreds times, with 1 hundred remaining; 1 hundred equals 10 tens, which, with 3 tens, equal 13 tens; 2 is contained in 13 tens 6 tens times, and 1 ten remaining; 1 ten equals 10 units, which, with 6 units, equal 16 units; 2 is contained in 16 units 8 units times. Hence the quotient is 268.	OPERATION 2)536 268

SOLUTION 2.—2 is contained in 5, 2 times, and 1 remaining; 2 is contained in 13, 6 times, and 1 remaining; etc.

RULE.—1. *Write the divisor at the left of the dividend; begin at the left hand, and divide each term of the dividend by the divisor, and write the quotient beneath.*

2. *If there is a remainder after any division, regard it as prefixed to the next term of the dividend, and divide as before. If any partial dividend is less than the divisor, prefix it to the next term, and write a cipher in the quotient.*

63. PROOF.—Multiply the quotient by the divisor, and add the remainder, if any, to the product.

(2.) 2)456	(3.) 2)736	(4.) 2)548	(5.) 2)374	(6.) 2)538
(7.) 3)735	(8.) 3)816	(9.) 3)522	(10.) 3)414	(11.) 3)738
(12.) 3)567	(13.) 3)513	(14.) 3)645	(15.) 3)765	(16.) 3)825
(17.) 4)512	(18.) 4)624	(19.) 4)732	(20.) 4)576	(21.) 4)824
(22.) 4)736	(23.) 4)816	(24.) 4)972	(25.) 4)608	(26.) 4)436
(27.) 5)615	(28.) 5)735	(29.) 5)645	(30.) 5)785	(31.) 5)840
(32.) 5)815	(33.) 5)935	(34.) 5)780	(35.) 5)765	(36.) 5)980

(37.)	(38.)	(39.)	(40.)	(41.)
6)834	6)738	6)654	6)774	6)864

(42.)	(43.)	(44.)	(45.)	(46.)
6)1476	6)3336	6)2514	6)3654	6)7236

(47.)	(48.)	(49.)	(50.)	(51.)
7)2569	7)4732	7)8456	7)9359	7)9870

Divide
52. 8256 by 8.
53. 7656 by 8.
54. 9576 by 8.
55. 9874 by 9.
56. 9756 by 9.
57. 9387 by 9.
58. 92565 by 9.

Divide
59. 72352 by 8.
60. 23769 by 9.
61. 73145 by 5.
62. 5882597 by 7.
63. 1101032 by 8.
64. 21820708 by 4
65. 6328476 by 9.

LONG DIVISION.

64. ***Long Division*** is the method of dividing when the partial dividends are written.

Case I.

65. When the divisor and quotient are each one figure.

1. Divide 7 by 2.

	OPERATION.
Solution 1.—2 is contained in 7 *three* times. We place the 3 at the right in the quotient, and multiply the divisor by it. 3 times 2 are 6, which we write under the 7. We then draw a line beneath, and subtract, and have 1 remaining.	2)7(3 6 — 1

Solution 2.—2 is contained in 7, 3 times; 3 times 2 are 6; 6 from 7 leaves 1; hence the quotient is 3, and 1 remaining.

(2.)	(3.)	(4.)	(5.)	(6.)	(7.)
2)5(	2)9(	2)10(	2)12(	2)15(	2)18(

(8.) 2)13(	(9.) 2)11(	(10.) 3)6(	(11.) 3)9(	(12.) 3)8(	(13.) 3)12(
(14.) 3)18(	(15.) 3)21(	(16.) 3)17(	(17.) 3)19(	(18.) 3)23(	(19.) 3)27(
(20.) 4)8(	(21.) 4)12(	(22.) 4)20(	(23.) 4)28(	(24.) 4)30(	(25.) 4)10(
(26.) 4)13(	(27.) 4)23(	(28.) 4)27(	(29.) 5)10(	(30.) 4)20(	(31.) 5)25(
(32.) 5)45(	(33.) 5)35(	(34.) 5)27(	(35.) 5)38(	(36.) 5)43(	(37.) 5)47(
(38.) 6)12(	(39.) 6)24(	(40.) 6)34(	(41.) 6)50(	(42.) 6)59(	(43.) 6)49(
(44.) 7)28(	(45.) 7)49(	(46.) 7)50(	(47.) 7)60(	(48.) 7)48(	(49.) 7)57(
(50.) 8)24(	(51.) 8)37(	(52.) 8)70(	(53.) 8)69(	(54.) 8)59(	(55.) 8)76(
(56.) 9)27(	(57.) 9)63(	(58.) 9)57(	(59.) 9)70(	(60.) 9)76(	(61.) 9)89(

CASE II.

66. When the divisor is one figure and the quotient is several figures.

1. Divide 867 by 3.

OPERATION.

```
3)867(289
  6
  --
  26
  24
  --
   27
   27
   --
```

SOLUTION 1.—3 is contained in 8 hundreds 2 hundreds times. 2 hundreds times 3 equal 6 hundreds. 6 hundreds from 8 hundreds leave 2 hundreds. 2 hundreds and 6 tens are 26 tens. 3 is contained in 26 tens 8 tens times. 8 tens times 3 are 24 tens. 24 tens from 26 tens leave 2 tens. 2 tens and 7 units are 27 units. 3 is contained in 27 units 9 times. 9 times 3 are 27. Subtracting, nothing remains. Hence, the quotient is 287.

SOLUTION 2.—3 is contained in 8, 2 times; 2 times 3 are 6; 6 from 8 leaves 2. Bring down the 6, and we have 26. 3 is contained in 26, 8 times; 8 times 3 are 24; 24 from 26 leaves 2. Bring down the 7, and we have 27. 3 is contained in 27, 9 times; 9 times 3 are 27, etc.

EXAMPLES FOR PRACTICE.

(2.) 2)36(	(3.) 2)58(	(4.) 2)54(	(5.) 2)92(	(6.) 2)97(
(7.) 3)576(	(8.) 3)465(	(9.) 3)723(	(10.) 3)873(	(11.) 3)675(
(12.) 4)852(	(13.) 4)764(	(14.) 4)932(	(15.) 4)576(	(16.) 4)748(
(17.) 5)735(	(18.) 5)850(	(19.) 5)975(	(20.) 5)745(	(21.) 5)835(
(22.) 6)732(	(23.) 6)846(	(24.) 6)924(	(25.) 6)972(	(26.) 6)834(
(27.) 7)784(	(28.) 7)798(	(29.) 7)833(	(30.) 7)966(	(31.) 7)959(
(32.) 8)896(	(33.) 8)936(	(34.) 8)944(	(35.) 8)976(	(36.) 8)992(

Divide
37. 37596 by 2.
38. 57672 by 3.
39. 78908 by 4.
40. 93546 by 6.
41. 73455 by 5.
42. 75448 by 8.
43. 45794 by 7.
44. 36783 by 9.
45. 487652 by 7.

Divide
46. 46542 by 3.
47. 785641 by 6.
48. 218030 by 8.
49. 51600 by 4.
50. 84507 by 7.
51. 61243 by 2.
52. 47065 by 5.
53. 31696 by 6.
54. 20040 by 9.

CASE III.

67. When the divisor is two or more figures.

1. Divide 442 by 13.

SOLUTION.—13 is contained in 44 tens 3 tens times; 3 tens times 13 equal 39 tens; 39 tens from 44 tens leave 5 tens, and bringing down the 2 units we have 52 units. 13 is contained in 52 units 4 times. 4 times 13 are 52; subtracting, nothing remains. Hence the quotient is 34.

OPERATION.

```
13)442(34
   39
   --
    52
    52
    --
```

NOTE.—With young pupils, abbreviate the explanation, as in the previous solutions.

RULE.—1. *Divide the number expressed by the fewest terms on the left of the dividend that will contain the divisor, and place the quotient on the right.*

2. *Multiply the divisor by this quotient; write the product under the partial dividend, and subtract, and to the remainder annex the next term of the dividend.*

3. *Divide as before, and thus continue until all the terms of the dividend have been used.*

4. *If any partial dividend will not contain the divisor, place a cipher in the quotient, annex the next term of the dividend, and proceed as before.*

68. PROOF.—Multiply the quotient by the divisor, and add the remainder, if any, to the product.

NOTES.—1. The pupils will notice that there are four operations: 1st, *Divide*, 2d, *Multiply*, 3d, *Subtract*, 4th, *Bring down*.

2. If when we multiply the product is greater than the partial dividends, the quotient figure is too large, and must be diminished.

3. When a remainder is equal to or greater than the divisor, the quotient figure is too small, and must be increased.

4. A final remainder may be set off by itself, or it may be written over the divisor and annexed to the quotient.

EXAMPLES FOR PRACTICE.

Divide
2. 364 by 11.
3. 780 by 12.
4. 312 by 13.
5. 322 by 14.
6. 570 by 15.
7. 752 by 16.
8. 425 by 17.
9. 594 by 18.
10. 608 by 19.
11. 945 by 21.
12. 2760 by 22.
13. 2852 by 23.
14. 3168 by 24.
15. 5575 by 25.
16. 6396 by 26.
17. 6777 by 27.
18. 10136 by 28.
19. 11948 by 29.
20. 19778 by 31.
21. 16864 by 32.
22. 10725 by 33.
23. 20808 by 34.
24. 7875 by 35.
25. 20616 by 36.
26. 41602 by 37.
27. 39790 by 38.
28. 48725 by 39.
29. 67314 by 41.
30. 82307 by 42.
31. 57256 by 43.
32. 49378 by 44.
33. 98716 by 45.
34. 60904 by 46.
35. 76704 by 47.

Divide
36. 62377 by 49.
37. 84309 by 57.
38. 92736 by 83.
39. 41875 by 123.
40. 1067500 by 500.
41. 1320594 by 561.
42. 1048788 by 468.
43. 932694 by 371.
44. 1011312 by 432.
45. 1459865 by 605.
46. 1612072 by 616.
47. 1704186 by 627.
48. 1519368 by 696.
49. 2103409 by 649.
50. 2815272 by 732.
51. 2592840 by 620.
52. 3564288 by 819.
53. 4227328 by 896.
54. 3516825 by 975.
55. 2802690 by 990.
56. 8901207 by 1449.
57. 9572160 by 1560.
58. 6192138 by 1653.
59. 3515772 by 1736.
60. 9876480 by 1976.
61. 24197460 by 2492.
62. 8231505 by 1905.
63. 13896225 by 2975.
64. 16084440 by 5058.
65. 23103465 by 6391.
66. 18356508 by 16074.
67. 576105376 by 78617.
68. 344943192 by 134376.
69. 1806147420 by 35805.

Case IV.

69. When ciphers are on the right of the divisor.

1. Divide 7654 by 500.

Solution.—We find how many times 5 *hundreds* is contained in 76 *hundreds* by dividing 76 by 5. It is contained 15 times, with a remainder of 1 hundred, which, with 54, equals 154	OPERATION. 5|00)76|54 15–154 or $15\frac{154}{500}$

Rule.—1. *Cut off the ciphers at the right of the divisor, and as many places from the right of the dividend.*

2. *Divide the remaining part of the dividend by the remaining part of the divisor; prefix the remainder to the figures cut off, for the true remainder.*

Note.—When the divisor, with the ciphers cut off, is greater than 12, we will of course divide by long division.

Divide
2. 189 by 50.
3. 487 by 60.
4. 985 by 80.
5. 1837 by 400.
6. 2572 by 1100.
7. 4783 by 1200.
8. 8725 by 1300.
9. 4687 by 1400.
10. 9876 by 1500.

Divide
11. 18732 by 1600.
12. 28732 by 1700
13. 19873 by 1900.
14. 25307 by 2100.
15. 40302 by 2500.
16. 87316 by 3400.
17. 92913 by 4600.
18. 31200 by 5100.
19. 8732000 by 12300.

PRACTICAL PROBLEMS.

Case I.

70. To divide a number by an equal part.

1. At 5 dollars each, how many sheep can you buy for 675 dollars?

Solution.—If 5 dollars will buy one sheep, 675 dollars will buy as many sheep as 5 is contained times in 675, which are 135. Hence, you can buy 135 sheep.	OPERATION. 5)675 135 Ans

2. At 12 dollars each, how many pigs can you buy for 3780 dollars? Ans. 315.

3. At 6 cents apiece, how many oranges can you buy for 354 cents? Ans. 59.

4. At 11 cents a quart, how many quarts of cherries can you buy for 1243 cents? Ans. 113.

5 In one pound there are 12 ounces; how many pounds in 1728 ounces?

6. In one minute there are 60 seconds; how many minutes in 12900 seconds?

7. How many cows can you buy for 2952 dollars, at the rate of 24 dollars each?

8. How many pounds of butter will 8100 cents buy, at the rate of 25 cents a pound?

9. There are 16 ounces in one pound; how many pounds in 5472 ounces?

10. In one bushel there are 32 quarts; how many bushels are there in 16182 quarts?

11. How many acres of land at 56 dollars an acre can you buy for 12152 dollars?

12. How long will it take a vessel to sail 6460 miles, at the rate of 68 miles a day?

13. The diameter of the earth is nearly 8000 miles; how long will it take a person to walk the distance, at the rate of 48 miles a day?

14. The circumference of the earth is nearly 25000 miles; how long will it take a person to walk it, at the rate of 50 miles a day?

15. The distance to the moon is 240,000 miles; how long would it take a balloon to reach it, moving at the rate of 75 miles an hour?

16. The sun is 95,000,000 miles from the earth; how long would it require a cannon-ball to reach it, moving at the rate of 48 miles a minute?

CASE II.

71. To divide a number into equal parts.

1. A man divides 387 dollars equally among 9 boys; how many dollars does each receive?

SOLUTION.—Each boy will receive as many dollars as 9 is contained times in 387, which are 43 dollars.

OPERATION.

$$9\overline{)387}$$
$$\overline{43}\text{ Ans.}$$

2. A lady divides 4860 dollars equally among 12 girls; how many dollars will each receive? Ans. 405.

3. A man earns 2639 dollars in 13 weeks; how much does he earn in one week? Ans. 203.

4. A man travels 1728 miles in 36 days; how far does he travel each day?

5. There are 25 pounds in a quarter; how many quarters are there in 34450 pounds?

6. There are 6468 cubic inches in 28 gallons; how many cubic inches in one gallon?

7. Sound moves 37060 feet in 34 seconds; how far will it move in 48 seconds?

8. There are 2583 gallons in 41 hogsheads; how many gallons in one hogshead?

9. If a road 57 miles long cost 7695 dollars, how much did it cost a mile?

10. A man gave 1725 dollars for cows worth 25 dollars each; how many cows did he buy?

11. How many bushels of oats at 56 cents a bushel can be bought for 13272 cents?

12. A man gave 1905 dollars for saddles worth 15 dollars each; how many did he buy?

13. A farmer sold 24 horses for 5640 dollars; how much did he receive apiece for them?

14. A farmer sold a lot of horses for 7685 dollars; how many did he sell, if he received 145 dollars each?

15. How many mules can you buy for 8332 dollars, at the rate of 184 dollars each?

PROBLEMS ON THE AREA OF STATES.

NEW ENGLAND STATES.

1. The area of Maine is 30000 square miles, and of New Hampshire 9280 square miles; how much larger is the former?

2. Vermont contains 9056 square miles, and Massachusetts 7800 square miles; how much larger is the former than the latter?

3. Rhode Island contains 1306 square miles, and Connecticut 4674 square miles; how much larger is Maine than both of these?

4. Which is larger, and how much, Maine or all the rest of the New England States? Which is larger, and how much, New Hampshire and Vermont together, or Massachusetts and Connecticut together?

MIDDLE STATES.

5. New York contains 47000 square miles, and New Jersey 8320 square miles; how much larger is the former?

6. Pennsylvania contains 46000 square miles, and Delaware 2120 square miles; how much larger is Pennsylvania?

7. Maryland contains 9356 square miles; how much larger is Maryland than New Jersey?

8. How much larger is Pennsylvania than New Jersey, Delaware, and Maryland all together?

9. How much larger are the Middle States than the New England States?

WESTERN STATES.

10. Ohio contains 39964 square miles, and Indiana 33809 square miles; how much larger is the former than the latter?

11. Michigan contains 56243 square miles, and Illinois 55405 square miles; how much larger is the former than the latter?

12. How much larger are Ohio and Michigan than Indiana and Illinois?

13. Wisconsin contains 53924 square miles, and Iowa 55045 square miles; how much larger is the latter than the former?

14. Missouri contains 67380 square miles, and Kentucky 37680 square miles; how much larger is Missouri?

15. Which would make the larger State, Wisconsin and Iowa, or Missouri and Kentucky?

16. California contains 189000 square miles, and Oregon 95000 square miles; which is the larger, and how much?

17. How much larger are the Western States than the New England and Middle States together?

SOUTHERN STATES.

18. Virginia contains 41352 square miles, and West Virginia 20000 square miles; how much larger is Virginia than West Virginia?

19. North Carolina contains 45000 square miles, and South Carolina 24500 square miles; how much larger is North Carolina than South Carolina?

20. Georgia contains 58000 square miles, and Louisiana 46431 square miles; how much larger is Georgia than Louisiana?

21. Alabama contains 50722 square miles, and Mississippi 47156 square miles; how much larger is Alabama than Mississippi?

22. Arkansas contains 52198 square miles, and Tennessee 45600; which is the larger, and how much?

23. Florida contains 59628 square miles, and Texas 237321 square miles; how much larger is Texas than Florida?

24. Which is larger, and how much, Texas, or all the other States taken together?

SECTION III.

UNITED STATES MONEY.

72. ***United States Money*** is the money of the United States.

TABLE.

10 mills ($m.$) equal	. . . .	1 cent, *c.*
10 cents	"	1 dime, *d.*
10 dimes	"	1 dollar, **$**.
10 dollars	"	1 eagle, E.

Coins are pieces of metal, stamped by the authority of the government, to be used as money.

GOLD COINS.			SILVER COINS.		
Eagle,	*value*	*$10*	*Dollar,*	*value*	*$1*
Double-eagle,	"	*20*	*Half-dollar,*	"	*50 c.*
Half-eagle,	"	*5*	*Quarter-dollar,*	"	*25 c.*
Quarter-eagle,	"	*2½*	*Dime,*	"	*10 c.*
Dollar,	"	*1*	*Half-dime,*	"	*5 c.*

NICKEL.		BRONZE.	
Five-cent, value	*5 c.*	*Cent, value*	*1 c.*

NUMERATION AND NOTATION.

73. The *dollar* is indicated by the symbol **$**. The eagle and dollar are read as a number of dollars: thus, 3 eagles and 5 dollars are read, 35 dollars.

74. The *dime* is one-tenth of a dollar, and is written to the right of the dollar and separated from it by a point, called a separatrix; thus, $3.4 represents 3 dollars and 4 dimes.

75. The *cent* is 1 tenth of a dime, or 1 hundredth of a dollar. It is written two places to the right of dollars; thus, $4.58 represents 4 dollars, 5 dimes, and 8 cents.

76. Dimes and cents are usually read as so many cents; thus, $7.45 is read, 7 dollars and 45 cents.

77. The *mill* is 1 tenth of a cent, and is written one place to the right of cents; thus, $5.475 is read, 5 dollars, 47 cents, and 5 mills.

PRACTICAL PROBLEMS.

EXAMPLES IN NUMERATION.

1. Write and read $24.75.

SOLUTION.—The pupil will write this upon the slate or black board, and say: This is read, 24 dollars, 7 dimes, and 5 cents; or. 24 dollars and 75 cents.

The pupil will write and read the following:

2. $14.25	6. $48.50	10 $105 076
3. $24.67	7. $50.06	11. $976.705
4. $19.84	8. $48.408	12. $350.035
5. $28.574	9. $96.004	13. $847.008

EXAMPLES IN NOTATION.

1. Write six dollars and twenty-five cents.

2. Write twenty-five dollars and thirty-six cents.
3. Write eight dollars, forty-five cents, and six mills.

4. Write twenty dollars, seventy-five cents, and two mills.
5. Write six eagles, seven dollars, and eighty-four cents.
6. Write four dollars, six dimes, and seven cents.
7. Write 25 dollars, five cents, and eight mills.

REDUCTION OF UNITED STATES MONEY.

78. ***Reduction*** consists in changing the denomination without changing the value. From the table we derive the following principles:

To reduce cents to mills, we multiply the cents by 10, *or annex* ONE *cipher.*

To reduce dollars to cents, we annex TWO *ciphers.*

To reduce dollars to mills, we annex THREE *ciphers.*

To reduce a number of dollars and cents to cents, we remove the decimal point; thus, $5.24 = 524 *cents.*

CASE I.

To reduce to lower terms.

1. Reduce 6 dollars to cents.

SOLUTION.—In 1 dollar there are 100 cents; hence, in 6 dollars there are 6 times 100 cents, or 600 cents; or we annex two ciphers.

OPERATION.
$6 = 600 cents

2. Reduce $18 to cents.
3. Reduce $24 to cents.
4. Reduce $385 to cents.
5. Reduce $27 to mills.
6. Reduce 85 cents to mills.
7. Reduce $5.47 to cents.
8. Reduce $27.05 to cents.
9. Change $9 607 to mills.

CASE II.

To reduce to higher terms.

79. From the table we have the following principles:

1. *To reduce cents to dollars, place the point* TWO *places from the right.*

2. *To reduce mills to dollars, place the point* THREE *places from the right.*

1. Reduce 2347 cents to dollars.

SOLUTION.—There are 100 cents in 1 dollar, and in 2347 cents there are as many dollars as 100 is contained times in 2347, which are $23.47; or we place the point two places from the right.

OPERATION.
2347 ÷ 100
= $23.47

2. Reduce 845 cents to dollars. Ans. $8.45.
3. Reduce 2835 cents to dollars. Ans. $28.35.

4. Reduce 46785 cents to dollars.
5. Reduce 7895 mills to dollars.
6. Reduce 27065 mills to dollars.
7. Reduce 4800 cents to dollars.
8. Reduce 9600 mills to dollars.

ADDITION OF UNITED STATES MONEY.

80. ***Addition of United States Money*** is performed as in simple numbers, according to the following

RULE.—1. *Write dollars under dollars, cents under cents, etc.*

2. *Add as in simple numbers, and place the separatrix between dollars and cents.*

1. Find the sum of $24.36, $96.58, and $75.42.

SOLUTION.—We write dollars under dollars and cents under cents, and commence at the right to add. 2 and 8 are 10, and 6 are 16 cents; which equals 6 cents and 1 dime; we write the 6 cents under the column of cents, and add the 1 dime to the next column, etc.

OPERATION.
$24.36
96.58
75.42
$196.36

2. Find the sum of $48.56, $39.46, $24.67, and $81.09.
3. Add $23.84, $97.36, $52.75, and $98.27.
4. Add $73.75, $48.56, $39.87, and $75.48.
5. Add $46.375, $97.283, $72.475, and $8.396.
6. Add $156.96, $284.076, $9.27, and $85.735.
7. A man bought a cow for $24.75, a horse for $150.50, a wagon for $287.75, and a carriage for $375.87; how much did he pay for all?
8. A merchant bought flour for $57.35, some calicc for $96.87, some cloth for $84.50, some boots for $52.87, and some muslin for $75.75; what did they all cost?
9. A tailor sold a coat for $34.75, a vest for $8.50, a cloak for $52.25, a pair of pants for $9.75, and some other things for $28.45; what did he receive for all?
10 I bought a table for $18.25, a looking-glass for

$25.75, a bedstead for $36.50, a bureau for $46.25; what did they all cost?

11. A owes $624.30, B owes $467.56, C owes $359.45, D owes $95.12, E owes $43.84, F owes $27.75, G owes $968.47, H owes $7.75; required the sum of their debts.

SUBTRACTION OF UNITED STATES MONEY.

81. ***Subtraction of United States Money*** is performed as in subtraction of simple numbers, according to the following

RULE.—1. *Write dollars under dollars, cents under cents, etc.*

2. *Subtract as in simple numbers, and place the separatrix between dollars and cents.*

1. Subtract $21.48 from $46.73.

	OPERATION.
SOLUTION.—We cannot subtract 8 cents from 3 cents, hence we add 10 cents to 3 cents, making 13 cents; 8 cents from 13 cents leave 5 cents. Now, since we added 10 cents, or 1 dime, to the minuend, we must add 1 dime to the 4 dimes, making 5 dimes: 5 dimes from 7 dimes leave 2 dimes, etc.	$46.73 27.48 $19.25

(2.)	(3.)	(4.)	(5.)
$78.25	$57.52	$96.43	$75.75
13.16	23.28	28.14	23.28

6. From $129.39 take $48.91.

7. Find the difference between $234.16 and $471.24.

8. A man bought a horse for $234.50, and sold it for $228.25; what did he lose?

9. A merchant bought cloth for $96.75, and sold it for $110.29; what did he gain?

10. A bought a farm for $3640.25, and sold it for $4000; what was the gain?

11. My house cost $3480.75, and I sold it for $4000.50; what did I gain?

12. My horse cost $240.50, and my carriage cost $386.25; I sold them for $680.50; what did I gain?

13. A merchant bought cloth for $325.50, muslin for $436.75, and flour for $625.80; he sold them all for $1300; how much did he lose?

14. I paid $4637.25 for a farm, paid $3675.25 for building a house, and $2896.87 for building a barn; I sold my property for $13000; how much did I gain?

15. I paid $246.75 for a horse, $325.45 for a mule, $42.25 for an ox, $37.50 for a cow; I sold them all for $603.50; what was the loss?

MULTIPLICATION OF UNITED STATES MONEY.

82. ***Multiplication of United States Money*** is performed like multiplication of simple numbers, according to the following

RULE.—*Multiply as in simple numbers, and place the separatrix between dollars and cents.*

1. Multiply $36.25 by 3.

SOLUTION.—Three times 5 cents are 15 cents, which equal 1 dime and 5 cents; we write the 5 cents, and reserve the 1 dime to add to the next product. 3 times 2 dimes are 6 dimes, and 6 dimes plus 1 dime are 7 dimes, etc.

OPERATION.

$$\begin{array}{r} \$36.25 \\ 3 \\ \hline \$108.75 \end{array}$$

Multiply	Multiply
2. $26.14 by 4.	7. $48.25 by 12.
3 $37.27 by 5.	8. $72.27 by 13.
4 $48.96 by 7.	9. $85.58 by 15.
5. $37.52 by 8.	10. $92.83 by 32.
6. $79.35 by 9.	11. $75.32 by 46.

12. If one yard of cloth cost $3.25, what cost 5 yards?

13. What will 12 horses cost at the rate of $150.75 apiece?

14. A man bought 27 oxen at the rate of $36.25 each; what did they cost?

15. A farmer sold 325 bushels of wheat at $1 25 a bushel; how much did he receive for it?

16. A miller sold 472 barrels of flour at $7.87 a barrel; how much did he receive for it?

17 A man bought 47 cows for $24.30 each, and sold them for $28.10 each; what was the gain?

18. A drover bought 247 horses for $130.75 each, and sold them for $180.30 each; what did he gain?

19. A farmer bought 327 acres of land at $76.25 an acre, and sold it at $92.50 an acre; what did he gain?

DIVISION OF UNITED STATES MONEY.

83. ***Division of United States Money*** is performed like division of simple numbers.

Case I.

84. To divide a number into equal parts.

Rule.—*Divide as in simple numbers, and place the separatrix between dollars and cents.*

1. Divide $7.32 in 3 equal parts, or find 1 third of it.

Solution.—1 third of 7 dollars is 2 dollars, and 1 dollar remaining; 1 dollar equals 10 dimes, which, added to 3 dimes, equal 13 dimes. 1 third of 13 dimes equals 4 dimes, and 1 dime remaining, etc.

OPERATION.

3)$7.32

$2.44 Ans.

2. Divide $9.24 into 4 equal parts.
3. Divide $7.25 into 5 equal parts.
4. Divide $17.22 into 6 equal parts.
5. If 7 pigs cost $36.75, what will one pig cost?
6. If 8 cows cost $172.80, what will one cow cost?
7. If 3 oxen cost $325.20, what will 5 oxen cost?
8. If 7 hens cost $3.15, what will 12 hens cost?
9. What cost 15 sheep, if 4 sheep cost $29.24?
10. What cost 25 pounds of butter, if 7 pounds cost $2.38?

11. What cost 34 acres of land, if 12 acres cost $5.04?
12. What cost 28 cows, if 35 cows cost 987 dollars?
13. What cost 75 oxen, if 38 oxen cost 1615 dollars?
14. What cost 234 hens, if 75 hens cost $25.50?

CASE II.

85. To divide one sum of money by another.

RULE.—*Reduce both sums to the same denomination, and divide as in simple numbers.*

1. Divide $736 by $4.

OPERATION.

SOLUTION.—Dividing as in simple numbers, we have 184.

4)736
184 Ans.

2. Divide $9600 by $16.
3. Divide 728 cents by 4 cents.
4. Divide 3625 cents by 5 cents.
5. Divide $26325 by 81 dollars.
6. At 24 dollars each, how many cows can you buy for 1344 dollars?
7. At 42 dollars each, how many oxen can be bought for $3276?
8. At $3.25 apiece, how many pigs can you buy for $120.25?
9. A earned $3.75 a day; how many days did he work to earn $78.75?
10. A drover paid $6972 for horses, at $145.25 apiece; how many did he buy?
11. How many cords of wood can you buy for $312, at $3.25 a cord?
12. William earned $3.25 a day, and paid 75 cents for board; in how many days would he save $912.50?
13. A merchant received $853.25 for a case of silk, including $1.25 cost of box. How many pieces of silk were in the case, if he received $53.25 apiece?

BILLS AND ACCOUNTS.

86. A ***Bill of Goods*** is a written statement of goods bought or sold, giving the place, date, names of buyer and seller, quantity, price, and entire cost.

An ***Account*** is a written statement of the debts and the credits of business transactions.

The party who owes is the *debtor;* the party who is owed is the *creditor.* A bill is made out by the following

RULE.—1. *Find the cost of the several items by multiplying the price of each by the quantity, and take the sum of the several products.*

2. *In an* ACCOUNT, *find the difference between the debit and credit amounts.*

Make out the following bills:

(1.) *Millersville, May 8, 1886.*

Mr. Harry Bowman,

Bought of HENRY MARTIN,

8	*yds. of muslin, at $0.27,*	*$*	
12	*" of cloth, " 2.37,*		
15	*" of silk, " 1.62,*		
	Amount due,	*$*	

(2.) *Lancaster, April 6, 1888.*

Theo. Miller,

Bought of DANIEL MOONEY,

24	*pairs boots, at $5.25,*	*$*	
37	*" gaiters, " 3.75,*		
45	*" slippers, " 1.37,*		
28	*" rubbers, " 1.25,*		
	Amount due,	*$*	

Received Payment,

Daniel Mooney.

(3.) *New York, Dec. 17, 1882.*

John J. Brooks,

Bought of CHARLES HOYT,

47	bbls. of flour, at $7.35,	$	
28	lbs. of beef, " 0.37,		
97	yds. of cloth, " 2.75,		
146	bu. of wheat, " 1.12,		
	Amount,	$	

Received Payment,

Charles Hoyt.

(4.)

John Smith, Dr.

1886.						
Jan.	1	To 75 lbs. of sugar, at $0.35,	$			
Feb.	5	" 47 yds. of cloth, " 3.25,				
		Cr.				
Jan.	7	By 75 bu. of corn, at $0.78,			$	
Feb.	2	" 83 bu. of apples, " 1.25,				
		Balance due,				

(5) *Philadelphia, April 1, 1880.*

Mr. Henry Farnam, Dr.

To Edwin Lamborn.

1880.						
Jan.	4.	To 145 bu. wheat, at $1.25,	$			
Jan.	10.	" 236 " rye, " 1.05,				
Jan.	20.	" 176 " oats, " 0.65,				
1860.		Cr.			$	
Jan.	3.	By 45 yds. cloth, at $3.65,				
Jan.	12.	" 72 " silk, " 2.12,				
Feb.	24.	" 80 " cassimere, " 1.75,				
		Balance due,				

Received Payment,

Edwin Lamborn.

BUSINESS PROBLEMS.

SUGGESTION.—Pupils will put these in the form of accounts, as on page 87.

1. A merchant sold a farmer 125 yards of calico, at 18 cents a yard, 150 yards of drilling, at 15 cents a yard, and bought of the farmer 225 bushels of oats, at 40 cents a bushel, and 90 bushels of rye, at $1.25 a bushel; which owes the other, and how much?

2. A mechanic sold a farmer a wagon for $56.50, two plows, at $7.50 each, and 6 wheel-barrows, at $5.25 each; and bought of the farmer 50 bushels of potatoes, at 75 cents a bushel, and 75 bushels of wheat, at 85 cents a bushel; which owes the other, and how much?

3. A farmer sold a merchant 4 cows, at $28.50 each, a yoke of oxen for $95, and 7 sheep, at $6.25 each; and took in payment 40 yards of carpet, at $2.25 a yard, 35 yards of cloth, at $3.25 a yard, and 58 yards of muslin, at 15 cents a yard; how much remains due?

4. A farmer bought of a mechanic, 2 wagons, at $76 each, 4 drags, at $6.50 each, 3 harrows, at $12.25 each; and sold him 45 bushels of apples, at 55 cents a bushel, 3 barrels of cider, at $5.25 a barrel, 28 bushels of corn, at 42 cents a bushel, and 3 cows, at $28.75 each; which owes the other, and how much?

5. A mechanic bought of a merchant

28 pounds of sugar, at 18cts. a pound,
36 pounds of rice, at 17cts. a pound,
45 yards of muslin, at 18cts. a yard,
28 yards of cloth, at $5.25 a yard,
37 barrels of flour, at $7.25 a barrel,

And sold him

4 wagons, at $75 each,
6 wagon-racks, at $13.50 each,
2 mowing-machines, at $157 each,
3 ox-yokes, at $6.75 each;

Which owes the other, and how much?

INTRODUCTION TO COMPOSITION, FACTORING, ETC.

Two or more numbers multiplied together *make* or produce another number, called the *product* of the others.

1. What numbers multiplied together *make* 4? 6? 8? 9? 12? 15? 18?

2. What numbers multiplied together *make* 10? 14? 16? 20? 24? 30?

3. What may we call the numbers whose product *makes* another number? Ans. The *makers* of the number.

4. What are the *makers* of 10? Of 30? Of 25? Of 27? Of 36? Of 48?

5. If the word *factor* means the same as *maker*, what may we call the makers of numbers? Ans. *Factors.*

6. What are the *factors* of 24? Of 21? Of 32? Of 33? Of 35? Of 40? Of 44? Of 60?

7. What may we call a number which is *composed* of the product of other numbers? Ans. A *composite number.*

8. Form composite numbers out of the factors 2 and 3; 3 and 4; 4 and 5; 2, 3, and 4; 3, 4, and 5.

9. Numbers which cannot be formed by the product of other numbers greater than 1 are called *prime numbers.*

10. Tell which of the following numbers are *prime* and which are *composite*: 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13.

11. Name three factors of 8; of 12; of 18; of 16; of 20; of 24; of 30; of 32; of 36; of 48.

12. When the factors of a number are *prime* numbers, what kind of factors shall we call them? Ans. *Prime factors.*

13. What are the prime factors of 10? Of 14? Of 12? Of 22? Of 30? Of 42? Of 70?

14. What would it seem natural to call the process of making composite numbers? Ans. *Composition.*

15. What would it seem natural to call the process of finding the factors of a number? Ans. *Factoring.*

16. How, then, shall we define the terms *Factor, Composite Number, Prime Number, Composition, Factoring?*

SECTION IV.

COMPOSITION AND FACTORING.

87. A ***Composite Number*** is one that can be produced by multiplying two or more numbers together, each of which is greater than a unit.

Thus, 6 is a composite number, since it can be produced by multiplying 3 and 2 together, each of which is greater than a unit.

A ***Prime Number*** is one that cannot be produced by multiplying two or more numbers together, each greater than a unit.

Thus, 2, 3, 5 and 7 are prime numbers, since they cannot be formed by the product of two numbers, each greater than a unit.

The ***Factors*** of a composite number are the numbers which, when multiplied together, will produce it.

Thus, 2 and 3 are the factors of 6, since 3 times 2 are 6; 4 and 3 are the factors of 12, since 4 times 3 are 12.

The ***Prime Factors*** of a number are the prime numbers which, when multiplied together, will produce it.

Thus, 2, 2 and 3 are the prime factors of 12.

MENTAL EXERCISES.

1. What numbers multiplied together will produce 6, 10, 12, 14, 15, 18, 20, 24, 33, 72, 84, 108, 156?
2. What are the factors of 10, 14, 15, 18, 21, 24, 25, 27, 28, 32, 33, 42, 55, 72, 96, 144, 216?
3. What prime numbers multiplied together will produce 6, 8, 12, 15, 16, 18, 20, 22, 24, 28, 35, 40, 56, 74, 125, 186?
4. What are the prime factors of 12, 18, 27, 36, 40, 64, 96, 132?

COMPOSITION.

88. ***Composition*** is the process of composing numbers out of their factors.

Thus, the production of 12 out of its factors, 3 and 4, is composition.

PRINCIPLE.—*Every composite number is equal to the product of its prime factors.*

1. Find the composite number whose factors are 2, 3 and 5.

OPERATION.

$$\begin{array}{r} 5 \\ 3 \\ \hline 15 \\ 2 \\ \hline 30 \end{array}$$

SOLUTION.—To find the composite number whose factors are 2, 3 and 5, find the product of these factors. 5 multiplied by 3 is 15, and 15 multiplied by 2 is 30. Hence the composite number is 30.

RULE.—*Find the product of all the factors.*

2. Find the composite number composed of the factors 3, 5 and 7. Ans. 105.

3. Find the composite number composed of the factors 7, 9, 17 and 89. Ans. 95319.

4. Find the composite numbers consisting of three equal factors, each being 5; each 7; each 15; each 59.

5. Find the composite numbers which have three equal factors, when each is 35; 87; 109; 163; 530; 657.

6. Find the composite numbers consisting of four equal factors, each being 3; 11; 17; 44; 54; 75; 153.

FACTORING.

89. ***Factoring*** is the process of finding the factors of composite numbers.

Thus, the finding of the factors, 3 and 4, of 12, is factoring.

PRINCIPLE.—*Every composite number is divisible by its prime factors.*

Thus, 15 is the product of its two prime factors, 3 and 5; hence 15 is divisible by 3 or 5.

1. What are the prime factors of 60?

OPERATION.

$$\begin{array}{r} 2)\underline{60} \\ 2)\underline{30} \\ 3)\underline{15} \\ 5 \end{array}$$

SOLUTION.—Dividing 60 by 2 we have a quotient of 30; dividing 30 by 2 we have a quotient of 15; dividing 15 by 3 we have a quotient of 5; hence 2, 2, 3 and 5 are the factors of 60, and since they are prime numbers they are the prime factors of 60.

RULE.—1. *Divide the given number by any prime number greater than* 1 *that will divide it.*

2. *Divide the quotient, if composite, in the same manner, and thus continue until the quotient is a prime number.*

3. *The divisors and the last quotients will be the prime factors required.*

Find the prime factors of

2. 48	6. 175	10. 475	14. 1200
3. 72	7. 270	11. 858	15. 7290
4. 81	8. 315	12. 1575	16. 29295
5. 108	9. 336	13. 8316	17. 341775

GREATEST COMMON DIVISOR.

90. A ***Divisor*** of a number is a number that will exactly divide it.

Thus, 4 is a divisor of 12, since it divides 12 without a remainder.

A ***Common Divisor*** of two or more numbers is a number that will exactly divide each of them.

Thus, 4 is a common divisor of 16 and 24, since it divides each of them without a remainder.

The ***Greatest Common Divisor*** of two or more numbers is the greatest number that will exactly divide each of them.

Thus, 18 is the greatest common divisor of 36 and 54, since it is the greatest number that will divide each of them without a remainder.

MENTAL EXERCISES.

1. Name some divisors of 8; 12; 18; 24; 36.
2. What factors are common to 8 and 12? 9 and 12? 20 and 30?
3. What divisors are common to 12 and 16? 18 and 24? 36 and 48?
4. What is the largest divisor common to 8 and 12? to 12 and 14? to 12 and 16? to 24, 36 and 72? to 25, 50 and 125?

PRINCIPLE.—*The greatest common divisor of two or more numbers equals the product of all the common prime factors of those numbers.*

1. Find the greatest common divisor of 24, 30 and 42.

SOLUTION.—The factors of 24 are 2, 3 and 4; the factors of 30 are 2, 3 and 5; the factors of 42 are 2, 3 and 7. The common factors of 24, 30 and 42 are 2 and 3; and the product of 2 and 3, or 6, is the greatest common divisor of 24, 30 and 42.

OPERATION.

$24 = 2 \times 3 \times 4$
$30 = 2 \times 3 \times 5$
$42 = 2 \times 3 \times 7$
$2 \times 3 = 6$

RULE.—*Resolve the numbers into their prime factors and take the product of all the common prime factors.*

Find the greatest common divisor

2. Of 30 and 36.
3. Of 60 and 90.
4. Of 44 and 66.
5. Of 96 and 84.
6. Of 175 and 245.
7. Of 12, 15 and 21.
8. Of 18, 24 and 36.
9. Of 36, 72 and 108.
10. Of 84, 126 and 210.
11. Of 556, 630 and 1638.

12. What is the length of the longest pole with which you can measure 126, 144 and 156 feet? Ans. 6 feet.

13. Three pieces of carpet, 1 yard wide, of 48, 64 and 80 yards, will exactly cover a parlor, if cut into the longest possible equal lengths. How long is the parlor? and how wide? Ans. Length, 16yds.; Width, 12yds.

LEAST COMMON MULTIPLE.

91. A ***Multiple*** of a number is one or more times that number.

Thus, 12 is a multiple of 4, since it is *three* times 4.

A ***Common Multiple*** of two or more numbers is a number which is a multiple of each of them.

Thus, 24 is a common multiple of 4 and 6, since it is a number of times each of them.

The ***Least Common Multiple*** of two or more numbers is the least number which is a multiple of each of them.

Thus, 12 is the least common multiple of 4 and 6, since it is the least number that is a number of times each of them.

MENTAL EXERCISES.

1. What number is a multiple of 3? of 4? of 5? of 6? of 7? of 8?
2. Name two multiples of 8; two multiples of 10; three multiples of 9; three multiples of 12.
3. What number is a multiple of both 4 and 6? 5 and 6? 6 and 8?
4. Name a common multiple of 3 and 4; 6 and 9; 8 and 12; 9 and 12.
5. Name the least common multiple of 4 and 6; of 4 and 8; of 6 and 8; of 8 and 10; of 9 and 12.

PRINCIPLE.—*The least common multiple of two or more numbers must contain all the factors of each number, and no other factors.*

1. Find the least common multiple of 6 and 15.

SOLUTION.—The prime factors of 6 are 2 and 3; hence the multiple must contain the factors 2 and 3. The factors of 15 are 3 and 5; hence the multiple must contain the additional factor, 5. The least common multiple, therefore, of 6 and 15 is $2 \times 3 \times 5$, or 30.

OPERATION.

$$6 = 2 \times 3$$
$$15 = 3 \times 6$$
$$\text{L. C. M.} = 2 \times 3 \times 5 = 30.$$

RULE.—*Resolve the numbers into their prime factors, and take the product of all the different factors, using each factor the greatest number of times it occurs in either number.*

NOTE.—If one number is a divisor of another, omit it.

Find the least common multiple

2. Of 12 and 15.
3. Of 15 and 18.
4. Of 16 and 18.
5. Of 48 and 72.
6. Of 27 and 135.
7. Of 6, 8 and 10.
8. Of 5, 9, 12 and 15.
9. Of 12, 15, 18 and 24.
10. Of 20, 84, 96 and 108.
11. Of 63, 105, 189 and 294.

12. At a sunday-school collection, four classes contributed equal amounts. In one class each member gave 5 cents; in another, 6 cents; in another, 8 cents; and in the fourth, 10 cents: what is the least sum with which this could happen? Ans. $1.20.

13. The piece-goods in a case of silk are to cut without waste into dress patterns of either 12, 15, 20 or 30 yards: what are the shortest lengths into which the piece-goods can be made? Ans. 60 yards.

CANCELLATION.

92. ***Cancellation*** is a process of shortening computations by rejecting common factors from the dividend and divisor.

PRINCIPLE.—*Cancelling a common factor from both dividend and divisor does not change the quotient.*

For if we divide 18 by 6, the quotient is 3; and also, if we resolve 18 and 6 into their factors, and cancel the common factor, 3, the quotient is then 3.

$$\frac{18}{6} = 3 \qquad \frac{\cancel{3} \times 6}{2 \times \cancel{3}} = 3$$

1. Divide 28 by 8.

SOLUTION.—Write the divisor, 8, under the dividend, 24. Resolve 28 into the factors, 4×7, and 8 into 2×4, and cancel the common factor, 4, in dividend and divisor, and we have 7 divided by 2, or $3\frac{1}{2}$

OPERATION.

$$\frac{28}{8} = \frac{\cancel{4} \times 7}{2 \times \cancel{4}} = \frac{7}{2} = 3\tfrac{1}{2}$$

RULE.—*Cancel the factors common to the dividend and divisor; then divide the product of the remaining factors of the dividend by the product of the remaining factors of the divisor.*

NOTE.—When a factor is cancelled, the unit, 1, takes its place, but need not be written, except in the quotient where there are no other factors.

2. Divide 48 by 30.
3. Divide 54 by 45.
4. Divide 72 by 63.
5. Divide 42 by 30.
6. Divide 90 by 50.
7. Divide 144 by 120.
8. Divide $4 \times 5 \times 6$ by 60.
9. Divide 70 by $2 \times 4 \times 5$.
10. Divide $4 \times 6 \times 8$ by $3 \times 5 \times 7$.
11. Divide $7 \times 9 \times 10$ by $3 \times 5 \times 7$.
12. Divide $8 \times 10 \times 12$ by $4 \times 5 \times 16$.
13. Divide $27 \times 12 \times 14$ by $9 \times 4 \times 7$.
14. Divide $72 \times 45 \times 140$ by $18 \times 24 \times 35$.
15. How many apples, at 2 cents each, can be got for 12 oranges, at 3 cents each? Ans. 18.
16. How many pigs, at 5 dollars each, can be obtained for 20 barrels of corn, at 3 dollars a barrel? Ans. 12.
17. How many bushels of oats, worth 55 cents a bushel can be exchanged for 44 bushels of rye, at 75 cents a bushel. Ans. 60.
18. A merchant exchanged 4 pieces of gros grain silk, each containing 50 yards, at 6 dollars a yard, for beaver cloth, worth 5 dollars a yard; how many pieces, each containing 30 yards, did he obtain? Ans. 8.

INTRODUCTION TO FRACTIONS.

IDEAS OF FRACTIONS.

1. If I cut an apple into *two equal parts*, what is *one* part called?

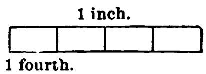
2. How many *halves* in one apple? How many halves in anything?

3. If I cut an apple into *three equal parts*, what is *one* of the parts called?

4. What are two of the parts called? How many thirds in one apple?

5. Which is larger, one half or one third? Two halves or two thirds?

6. If I divide an inch into four equal parts, what is one part called? What are 2, 3, 4 parts called?



7. How many fourths in 1 inch? How many fourths in half an inch?

8. To fracture is to break or divide into parts. These equal parts are called ***Fractions.*** What, then, is a fraction? Name some fractions.

NOTE.—With lines on the board, pieces of paper, or some similar objects, let the teacher develop the idea of fractions until they are clearly understood.

WRITING FRACTIONS.

1. We write 1 *half* thus, $\frac{1}{2}$; we write 1 *third* thus, $\frac{1}{3}$; we write 1 *fourth* thus, $\frac{1}{4}$; 1 *fifth* thus, $\frac{1}{5}$, etc.

2. How shall we write 2 halves? 2 thirds? 2 fourths? 3 fourths? 2 fifths? 1 sixth? 2 sixths? etc.

3. Write the following fractions:

1 fifth,	2 sixths,	1 seventh,	1 eighth,	1 ninth,
3 fifths,	3 sixths,	2 sevenths,	2 eighths,	4 ninths,
4 fifths,	5 sixths,	5 sevenths,	7 eighths,	8 ninths,

4. Read the following fractions:

$\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$, $\frac{4}{7}$, $\frac{5}{8}$, $\frac{7}{9}$, $\frac{3}{10}$, $\frac{6}{11}$, $\frac{5}{12}$, $\frac{7}{13}$.

5. The number written below the line is called the *denominator* of the *fraction*, because it gives the *name* to the parts.

6. The number written above the line is called the *numerator*, because it *numbers* the parts.

7. Name the numerator and denominator in the following fractions: $\frac{3}{4}$, $\frac{4}{5}$, $\frac{2}{6}$, $\frac{7}{8}$, $\frac{5}{7}$, $\frac{8}{9}$, $\frac{11}{12}$, $\frac{12}{13}$.

KINDS OF FRACTIONS.

1. In 2 *apples* what is the unit? Ans. One *apple*. In 3 oranges what is the unit?

2. How many thirds in *one* apple? In *two* apples? How many 4ths in 1 apple? In 2 apples?

3. To get 4 thirds how many apples must we cut into thirds? To get 6 fourths how many apples must we cut into fourths?

4. Which is greater, $\frac{4}{3}$ or *one?* $\frac{5}{4}$ or *one?* $\frac{6}{4}$ or *one?* $\frac{7}{5}$ or *one?* $\frac{9}{8}$ or *one?*

5. Fractions that are greater than a unit are called *improper fractions*, because they were not thought to be properly fractions.

6. Fractions less than a unit are called *proper fractions*, because they are properly a part of a unit.

7. Tell which are proper and which are improper fractions in the following: $\frac{3}{4}$, $\frac{5}{4}$, $\frac{4}{5}$, $\frac{7}{5}$, $\frac{8}{6}$, $\frac{5}{7}$, $\frac{4}{8}$, $\frac{11}{7}$, $\frac{7}{11}$.

8. If we unite, or "mix together," an integer, and a fraction, as 4 and $\frac{2}{3}$—thus, $4\frac{2}{3}$—we have what is called a *mixed number*.

9. Read the following mixed numbers: $2\frac{1}{2}$, $3\frac{2}{3}$, $4\frac{3}{4}$, $3\frac{2}{5}$, $5\frac{3}{6}$, $7\frac{5}{8}$, $8\frac{3}{7}$.

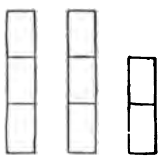
10. If we combine one fraction with another, forming "a compound" of two or more fractions—as, $\frac{1}{2}$ of $\frac{2}{3}$—we have what is called a *compound fraction*.

TREATMENT OF FRACTIONS.

Pupils may be aided in understanding the different cases of fractions by concrete illustrations.

1. How many thirds in $2\frac{2}{3}$?

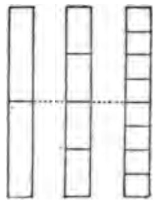
ILLUSTRATION.—We see that each unit is 3 thirds, and 2 units are 6 thirds; and these, with the 2 thirds of the third line, make 8 thirds in all.



$2\frac{2}{3} = \frac{8}{3}$

2. How many thirds in $3\frac{1}{3}$? Fourths in $2\frac{3}{4}$? Fifths in $3\frac{4}{5}$? Sixths in $4\frac{5}{6}$?

3. In $\frac{1}{2}$ how many fourths? How many 8ths?

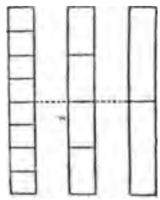


$\frac{1}{2} = \frac{2}{4} = \frac{4}{8}.$

ILLUSTRATION.—We see by the lines divided into halves, fourths, and eighths, that $\frac{1}{2}$ equals $\frac{2}{4}$, and also $\frac{4}{8}$.

4. In $\frac{2}{3}$ how many 6ths? How many 9ths? In $\frac{3}{4}$ how many 8ths? How many 12ths? In $\frac{4}{5}$ how many 10ths?

5. In $\frac{4}{8}$ how many 4ths? How many halves?



$\frac{4}{8} = \frac{2}{4} = \frac{1}{2}$

ILLUSTRATION.—We see by the lines divided into 8ths, 4ths, and halves, that $\frac{4}{8}$ equals $\frac{2}{4}$, and also $\frac{1}{2}$.

NOTE.—We can also illustrate with the lines in a horizontal position.

6. In $\frac{8}{12}$ of an inch how many 6ths of an inch? How many 3ds?

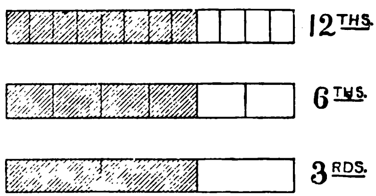
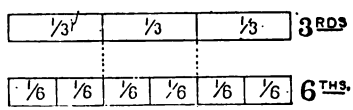


ILLUSTRATION.—We see by the illustration that $\frac{8}{12}$ is equal to $\frac{4}{6}$, and also to $\frac{2}{3}$.

7. Reduce $\frac{3}{6}$ to halves; $\frac{4}{6}$ to thirds; $\frac{6}{8}$ to fourths; $\frac{5}{10}$ to halves; $\frac{6}{9}$ to thirds; $\frac{3}{4}$ to 12ths.

8. What is $\frac{1}{2}$ of $\frac{1}{3}$ of an inch?

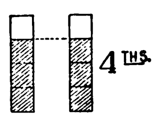


$\frac{1}{2}$ of $\frac{1}{3} = \frac{1}{6}$.

ILLUSTRATION.—To find $\frac{1}{2}$ of $\frac{1}{3}$ we divide one-third into 2 equal parts. If we divide each third of an inch into 2 equal parts, there will be *six* parts in all, and each part will be *one-sixth* of an inch. Hence we see that $\frac{1}{2}$ of $\frac{1}{3}$ of an inch is $\frac{1}{6}$ of an inch.

9. What is $\frac{1}{2}$ of $\frac{1}{2}$? $\frac{1}{2}$ of $\frac{1}{4}$? $\frac{1}{2}$ of $\frac{1}{5}$? $\frac{1}{3}$ of $\frac{1}{2}$? $\frac{1}{3}$ of $\frac{1}{4}$? $\frac{1}{3}$ of $\frac{2}{4}$; $\frac{1}{3}$ of $\frac{4}{5}$?

10. How many are $\frac{3}{4}$ of 2?



$\frac{3}{4}$ of $2 = \frac{6}{4}$.

ILLUSTRATION.—Divide each of the figures into fourths and take 3 fourths of both figures, and we have 6 fourths in all.

11. How many are $\frac{2}{3}$ of 2? $\frac{2}{3}$ of 3? $\frac{3}{4}$ of 3? $\frac{3}{4}$ of 4? $\frac{2}{3}$ of $2\frac{1}{2}$? $\frac{3}{4}$ of $3\frac{2}{3}$?

12. Divide $\frac{3}{4}$ by $\frac{2}{3}$.

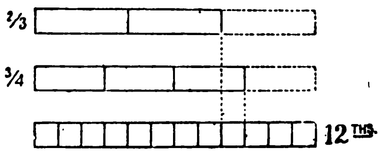


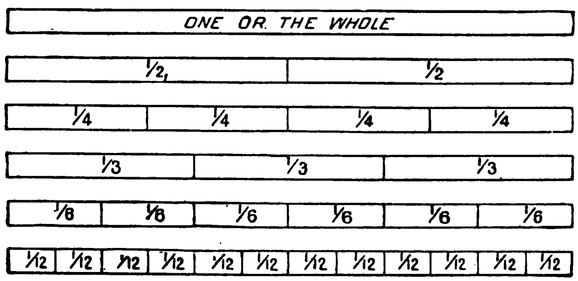
ILLUSTRATION.—We see by the figure that $\frac{2}{3}$ equals $\frac{8}{12}$ and $\frac{3}{4}$ equals $\frac{9}{12}$, and 8 *twelfths* are contained in 9 *twelfths* as often as 8 is contained in 9, which is $\frac{9}{8}$ times, or $1\frac{1}{8}$ times.

13. Divide $\frac{3}{4}$ by $\frac{2}{3}$; $\frac{4}{5}$ by $\frac{2}{3}$; $\frac{5}{6}$ by $\frac{2}{3}$; $\frac{3}{4}$ by $\frac{5}{6}$; $\frac{5}{6}$ by $\frac{3}{4}$; $\frac{5}{8}$ by $\frac{3}{4}$; $\frac{5}{8}$ by $\frac{1}{2}$; $\frac{3}{4}$ by $\frac{1}{2}$.

NOTE.—In a similar manner all the operations of fractions may be illustrated. We can use *lines*, *squares*, *circles*, pieces of *paper*, or pieces of *wood* prepared for the purpose.

These illustrations are, however, to be regarded only as stepping-stones to the subject. The pupil must be led to reason abstractly on numbers, or he will never understand the subject properly.

The following diagram indicates a convenient illustration of the relation of fractions, and may be used to explain most of the principles:



SECTION V.

COMMON FRACTIONS.

93. A ***Fraction*** is a number of equal parts of a unit; as one half, two thirds, etc.

94. A fraction is expressed by figures with a line between; thus, $\frac{2}{3}$ expresses 2 *thirds*.

95. The number denoted by the figure below the line is called the *denominator;* it shows the number of equal parts into which the unit is divided.

96. The number denoted by the figure above the line is the *numerator;* it shows the number of equal parts considered.

97. A ***Proper Fraction*** is one whose value is less than a unit; as $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{7}$, etc.

98. An ***Improper Fraction*** is one whose value is equal to or greater than a unit; as $\frac{4}{4}$, $\frac{8}{5}$, $\frac{21}{8}$, etc.

99. A ***Compound Fraction*** is a fraction of a fraction; as $\frac{1}{2}$ of $\frac{2}{3}$.

100. A ***Mixed Number*** consists of a whole number and a fraction; as $2\frac{1}{2}$, $5\frac{3}{8}$, etc.

TO TEACHERS.—Give pupils a clear idea of a fraction by dividing some object, as an *apple*, by *lines* upon the blackboard, etc. For illustrations of each case, see *Introduction to Fractions.*

MENTAL EXERCISES.

1. What is one-half?

Ans. One-half of any thing is one of the two equal parts of it

What is

2. One-third?
3. One-fourth?
4. One-fifth?
5. One-sixth?

What is

6. One-seventh?
7. One-eighth?
8. One-tenth?
9. One-twelfth?

1. What is two-thirds?

Ans. Two-thirds of any thing is two of the three equal parts of it.

What is

2. Two-fourths?
3. Three-fourths?
4. Two-fifths?
5. Three-fifths?

What is

6. Four-fifths?
7. Two-sixths?
8. Three-sevenths?
9. Four-ninths?

1. What is $\frac{1}{2}$ of 6?

Ans. $\frac{1}{2}$ of 6 is 3, since 2 times 3 are 6.

2. Find $\frac{1}{2}$ of 8.
3. Find $\frac{1}{3}$ of 12.
4. Find $\frac{1}{4}$ of 16.
5. Find $\frac{2}{3}$ of 15.
6. Find $\frac{3}{4}$ of 20.
7. Find $\frac{4}{5}$ of 30.

NUMERATION AND NOTATION.

Read the following fractions:

1. $\frac{5}{6}$; $\frac{6}{7}$.
2. $\frac{7}{9}$; $\frac{3}{5}$.
3. $\frac{3}{8}$; $\frac{6}{11}$.
4. $\frac{11}{13}$; $\frac{8}{10}$.
5. $\frac{10}{20}$; $\frac{7}{15}$.
6. $5\frac{3}{7}$; $11\frac{5}{9}$.

Write the following fractions:

1. Two-thirds.
2. Four-fifths.
3. Five-sevenths.
4. Eight-tenths.
5. Seven-ninths.
6. Eleven-fifteenths.

1. Analyze the fraction $\frac{3}{4}$.

SOLUTION.—In the fraction $\frac{3}{4}$, the denominator, 4, shows that the unit is divided into 4 equal parts, and the numerator, 3, shows that 3 of these parts are taken.

Analyze the following:

2. $\frac{2}{3}$; $\frac{4}{7}$.
3. $\frac{5}{6}$; $\frac{4}{5}$.
4. $\frac{3}{7}$; $\frac{2}{11}$.
5. $\frac{4}{8}$; $\frac{3}{11}$.
6. $\frac{7}{9}$; $\frac{8}{11}$.
7. $\frac{12}{13}$; $\frac{8}{14}$.
8. $\frac{9}{15}$; $\frac{7}{18}$.
9. $\frac{13}{21}$; $\frac{18}{23}$.
10. $\frac{21}{31}$; $\frac{34}{44}$.

PRINCIPLES OF FRACTIONS.

100$\frac{1}{2}$. We will now solve a number of problems, and derive some of the principles of fractions.

1. Multiply the numerator of $\frac{3}{5}$ by 2.

SOLUTION.—Multiplying the numerator of $\frac{3}{5}$ by 2, we have 6 *fifths*, which is 2 times as great as 3 *fifths*. Hence the following

OPERATION.
$\frac{3}{5} \times 2 = \frac{6}{5}$

PRINCIPLE I.—*Multiplying the numerator of a fraction by any number multiplies the fraction by that number.*

Multiply the fraction

2. $\frac{3}{4}$ by 5. Ans. $\frac{15}{4}$.
3. $\frac{5}{6}$ by 7.
4. $\frac{12}{15}$ by 8.
5. $\frac{13}{17}$ by 11.

Multiply the fraction

6. $\frac{13}{15}$ by 14.
7. $\frac{17}{19}$ by 18.
8. $\frac{18}{23}$ by 17.
9. $\frac{19}{21}$ by 20.

1. Divide the numerator of $\frac{4}{5}$ by 2.

SOLUTION.—Dividing the numerator of $\frac{4}{5}$ by 2, we have 2 *fifths*, which is 1 half of 4 *fifths*. Hence the following

OPERATION.
$\frac{4}{5} \div 2 = \frac{2}{5}$

PRINCIPLE II.—*Dividing the numerator of a fraction by any number divides the fraction by that number.*

Divide the fraction

2. $\frac{6}{7}$ by 3. Ans. $\frac{2}{7}$.
3. $\frac{8}{9}$ by 4.
4. $\frac{10}{11}$ by 5.
5. $\frac{14}{17}$ by 7.

Divide the fraction

6. $\frac{12}{13}$ by 4.
7. $\frac{18}{28}$ by 9.
8. $\frac{144}{151}$ by 12.
9. $\frac{256}{321}$ by 32.

1. Multiply the denominator of $\frac{3}{4}$ by 2.

SOLUTION.—Multiplying the denominator by 2, we have 3 *eighths*, which is one-half as much as 3 *fourths*, since *eighths* are only *half* as large as *fourths*. Hence the following

OPERATION.
$\frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$

PRINCIPLE III.—*Multiplying the denominator of a fraction by any number divides the fraction by that number.*

Divide the fraction

2. $\frac{2}{3}$ by 4. Ans. $\frac{2}{12}$.
3. $\frac{11}{12}$ by 7. Ans. $\frac{11}{84}$.
4. $\frac{3}{4}$ by 5.
5. $\frac{18}{19}$ by 7.
6. 7 [illegible] by 3.

Divide the fraction

7. $\frac{15}{22}$ by 8
8. $\frac{5}{8}$ by 6.
9. $\frac{13}{16}$ by 12.
10. $\frac{9}{10}$ by 11.
11. $\frac{15}{19}$ by 13.

1. Divide the denominator of $\frac{3}{4}$ by 2.

SOLUTION.—Dividing the denominator by 2, we have 3 halves, and 3 halves is twice as great as 3 fourths, since *halves* are *twice* as large as *fourths*. Hence the following

PRINCIPLE IV.—*Dividing the denominator of a fraction by any number multiplies the fraction by that number.*

Multiply, by dividing the denominator,

2. $\frac{3}{8}$ by 2.
3. $\frac{13}{12}$ by 6.
4. $\frac{5}{6}$ by 3.
5. $\frac{17}{18}$ by 9.
6. $\frac{7}{8}$ by 4.
7. $\frac{19}{21}$ by 7.
8. $\frac{9}{10}$ by 5.
9. $\frac{29}{48}$ by 12.
10. $\frac{15}{18}$ by 6.
11. $\frac{41}{76}$ by 19.
12. $\frac{11}{20}$ by 10.
13. $\frac{91}{144}$ by 36.

1. Multiply both numerator and denominator of $\frac{2}{3}$ by 2.

SOLUTION.—Multiplying both numerator and denominator by 2, we have $\frac{4}{6}$; and this equals $\frac{2}{3}$, since we both multiplied and divided $\frac{2}{3}$ by 2, and hence did not change its value. Hence we have the following

OPERATION.

$\frac{2}{3} \times \frac{2}{2} = \frac{4}{6}$

PRINCIPLE V.—*Multiplying both numerator and denominator of a fraction by the same number does not change the value of the fraction.*

2 Multiply both numerator and denominator of $\frac{3}{4}$ by 3; $\frac{4}{5}$ by 6; $\frac{3}{7}$ by 5; $\frac{8}{9}$ by 3; $\frac{11}{12}$ by 9; $\frac{13}{14}$ by 6.

1. Divide both numerator and denominator of $\frac{4}{6}$ by 2.

SOLUTION.—Dividing both numerator and denominator by 2, we have $\frac{2}{3}$; and this equals $\frac{4}{6}$, since we both divided and multiplied $\frac{4}{6}$ by 2, and hence did not change its value. From this we have

OPERATION.

$\frac{4}{6} \div \frac{2}{2} = \frac{2}{3}$

PRINCIPLE VI.—*Dividing both numerator and denominator by the same number does not change the value of the fraction.*

2. Divide both numerator and denominator of $\frac{6}{8}$ by 2; $\frac{8}{12}$ by 4; $\frac{9}{12}$ by 3· $\frac{10}{12}$ by 2; $\frac{16}{20}$ by 4; $\frac{20}{30}$ by 10: $\frac{30}{45}$ by 15.

Reduce to lowest terms

2. $\frac{6}{9}$, $\frac{14}{21}$. Ans. $\frac{2}{3}$
3. $\frac{8}{12}$, $\frac{12}{18}$. Ans. $\frac{2}{3}$.
4. $\frac{10}{12}$, $\frac{25}{30}$ Ans. $\frac{5}{6}$.
5. $\frac{16}{24}$, $\frac{18}{27}$.
6. $\frac{16}{28}$, $\frac{48}{84}$.

Reduce to lowest terms

7. $\frac{24}{40}$, $\frac{27}{45}$. Ans. $\frac{3}{5}$.
8. $\frac{70}{80}$, $\frac{84}{96}$.
9. $\frac{45}{50}$, $\frac{108}{120}$.
10. $\frac{99}{108}$, $\frac{121}{132}$.
11. $\frac{96}{104}$, $\frac{144}{156}$.

Case V.

108. To reduce compound fractions to simple.

1. What are $\frac{2}{3}$ of $\frac{4}{5}$?

Solution.—$\frac{1}{3}$ of $\frac{4}{5}=\frac{4}{15}$, since multiplying the denominator of a fraction by 3 divides the fraction by 3; and if $\frac{1}{3}$ of $\frac{4}{5}=\frac{4}{15}$, $\frac{2}{3}$ of $\frac{4}{5}$ equals 2 times $\frac{4}{15}$, which are $\frac{8}{15}$. From this solution we have the following

OPERATION.

$$\frac{2}{3}\times\frac{4}{5}=\frac{2\times4}{3\times5}=\frac{8}{15}$$

Rule.—*Multiply the numerators together, and the denominators together.*

What is

2. $\frac{3}{4}$ of $\frac{7}{8}$? Ans. $\frac{21}{32}$.
3. $\frac{5}{6}$ of $\frac{7}{9}$?
4. $\frac{4}{7}$ of $\frac{18}{15}$?
5. $\frac{8}{9}$ of $\frac{11}{12}$?
6. $\frac{5}{8}$ of $\frac{9}{10}$?

What is

7. $\frac{11}{12}$ of $\frac{16}{33}$?
8. $\frac{3}{4}$ of $\frac{5}{6}$ of $\frac{7}{8}$?
9. $\frac{4}{7}$ of $\frac{9}{10}$ of $\frac{14}{16}$?
10. $\frac{2}{5}$ of $\frac{7}{8}$ of $\frac{17}{18}$?
11. $\frac{2}{3}$ of $\frac{6}{7}$ of $\frac{14}{36}$?

12. A had $\frac{4}{5}$ of a ton of hay, and sold his neighbor $\frac{2}{3}$ of it; how much did he sell?

Solution.—If A had $\frac{4}{5}$ of a ton of hay, and sold his neighbor $\frac{2}{3}$ of it, he sold his neighbor $\frac{2}{3}$ of $\frac{4}{5}$ of a ton, which is $\frac{8}{15}$ of a ton.

OPERATION.

$\frac{2}{3}\times\frac{4}{5}=\frac{8}{15}$ Ans.

13. A boy picked $\frac{5}{6}$ of a bushel of strawberries, and sold $\frac{2}{3}$ of them; how many did he sell? Ans. $\frac{10}{18}$, or $\frac{5}{9}$.

14. A man had $\frac{5}{6}$ of a bushel of barley, and sold $\frac{3}{4}$ of it; how much did he sell? Ans. $\frac{5}{8}$.

15. A little girl had $\frac{7}{8}$ of a melon, and gave her brother $\frac{4}{7}$ of it; how much did her brother receive? Ans. $\frac{1}{2}$.

16. Says Jennie to Kate, My father owns $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ of a ship; what part of the ship did he own? Ans. $\frac{2}{5}$.

COMMON DENOMINATOR.

109. Fractions have a *Common Denominator* when they have the same number for a denominator.

1. Reduce $\frac{3}{4}$ and $\frac{4}{5}$ to a common denominator.

SOLUTION.—Multiplying both numerator and denominator of $\frac{3}{4}$ by 5, the denominator of $\frac{4}{5}$, we have $\frac{15}{20}$; and multiplying both numerator and denominator of $\frac{4}{5}$ by 4, the denominator of $\frac{3}{4}$, we have $\frac{16}{20}$; and this makes the fractions have the same denominator; hence the following

OPERATION.

$$\frac{3}{4} = \frac{3 \times 5}{4 \times 5} = \frac{15}{20}$$

$$\frac{4}{5} = \frac{4 \times 4}{5 \times 4} = \frac{16}{20}.$$

RULE.—*Multiply both numerator and denominator of each fraction by the denominators of the other fractions.*

For L. C. Denom., *Find the least common multiple of the denominators, divide this by the denominator of each fraction, and multiply both terms of the fraction by the quotient.*

Reduce to a common denominator

2 $\frac{2}{3}$ and $\frac{4}{5}$. Ans. $\frac{10}{15}$, $\frac{12}{15}$.
3. $\frac{4}{5}$ and $\frac{5}{6}$. Ans. $\frac{24}{30}$, $\frac{25}{30}$.
4. $\frac{7}{8}$ and $\frac{4}{5}$.
5. $\frac{3}{4}$ and $\frac{2}{7}$
6. $\frac{9}{10}$ and $\frac{8}{11}$.
7. $\frac{12}{13}$ and $\frac{13}{14}$.
8. $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{5}{6}$.
9. $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{4}{5}$.
10. $\frac{4}{5}$, $\frac{5}{6}$, and $\frac{6}{7}$.
11. $\frac{5}{6}$, $\frac{6}{7}$, and $\frac{7}{8}$.

ADDITION OF FRACTIONS.

110. ***Addition of Fractions*** is the process of finding the sum of two or more fractions.

CASE I.

To add when the denominators are alike.

1. What is the sum of $\frac{2}{5}$ and $\frac{4}{5}$?

SOLUTION.—2 *fifths* plus 4 *fifths* equals 6 *fifths*, which equals $1\frac{1}{5}$.

OPERATION.

$$\frac{2}{5} + \frac{4}{5} = \frac{6}{5} = 1\frac{1}{5}$$

2. What is the sum of $\frac{3}{6}$ and $\frac{2}{6}$? Ans. $\frac{5}{6}$.
3. What is the sum of $\frac{5}{6}$ and $\frac{4}{6}$? Ans. $\frac{9}{6}$, or $1\frac{1}{2}$
4 What is the sum of $\frac{5}{8}$ and $\frac{7}{8}$? Ans. $1\frac{1}{2}$.
5. What is the sum of $\frac{7}{9}$ and $\frac{8}{9}$? Ans. $\frac{15}{9}$, or $1\frac{2}{3}$.

6. Mary had $\frac{2}{5}$ of a dollar and Sarah had $\frac{4}{5}$ of a dollar; how much did they both have?

7. Lucy gave me $\frac{3}{6}$ of a peach, and Fanny gave me $\frac{5}{6}$ of a peach; how much did I receive? Ans. $1\frac{1}{3}$.

8. George and Susie had each $\frac{7}{8}$ of a pine-apple; how much had they together? Ans. $1\frac{3}{4}$.

9. If I walk $\frac{5}{6}$ of a mile and ride $\frac{4}{6}$ of a mile, how far do I go? Ans. $1\frac{1}{2}$ mile.

10. A had $\frac{3}{8}$ of a dollar, B had $\frac{7}{8}$ of a dollar, and C had $\frac{6}{8}$ of a dollar; how much had they all?

Case II.

To add when the denominators are unlike.

1. What is the sum of $\frac{2}{3}$ and $\frac{3}{4}$?

SOLUTION.—We first reduce the fractions to a common denominator: $\frac{2}{3}=\frac{8}{12}$; $\frac{3}{4}=\frac{9}{12}$; 8 *twelfths* plus 9 *twelfths* are 17 *twelfths*. Hence $\frac{2}{3}+\frac{3}{4}=\frac{17}{12}$. From this we have the following

OPERATION.

$$\frac{2}{3}+\frac{3}{4}=$$
$$\frac{8}{12}+\frac{9}{12}=\frac{17}{12}$$

RULE.—*Reduce the fractions to a common denominator; add the numerators, and place the sum over the common denominator.*

NOTE.—Reduce each fraction to its lowest terms before reducing to a common denominator, and also the result after addition.

Find the sum of

2. $\frac{2}{3}$ and $\frac{2}{5}$. Ans. $\frac{16}{15}$.
3. $\frac{3}{4}$ and $\frac{4}{5}$. Ans. $\frac{31}{20}$.
4. $\frac{3}{4}$ and $\frac{5}{6}$.
5. $\frac{4}{5}$ and $\frac{6}{7}$.
6. $\frac{3}{7}$ and $\frac{5}{8}$.
7. $\frac{2}{9}$ and $\frac{7}{10}$
8. $\frac{4}{5}$ and $\frac{8}{9}$.

Find the sum of

9. $\frac{7}{8}$ and $\frac{6}{7}$. Ans. $\frac{97}{56}$.
10. $\frac{3}{9}$ and $\frac{7}{10}$. Ans. $\frac{31}{30}$.
11. $\frac{4}{6}$ and $\frac{12}{14}$.
12. $\frac{10}{12}$ and $\frac{18}{20}$.
13. $\frac{12}{21}$ and $\frac{19}{20}$.
14. $\frac{16}{17}$ and $\frac{18}{24}$.
15. $\frac{19}{20}$ and $\frac{21}{22}$.

16. A has $\frac{2}{3}$ of a pie, and B $\frac{3}{4}$ of a pie; how much have they both?

17. B having $\frac{3}{4}$ of a ton of hay bought $\frac{4}{5}$ of a ton; how much had he then?

18. Henry owned $\frac{2}{7}$ of a vessel, and bought $\frac{3}{9}$ of the vessel; how much did he then own?

19. A had $7\frac{3}{4}$ dollars, and B gave him $8\frac{2}{3}$ dollars; how many had he then?

NOTE.—Add the 7 and 8, and then add $\frac{3}{4}$ and $\frac{2}{3}$; $8 + 7 = 15$; $\frac{3}{4} + \frac{2}{3} = \frac{9}{12} + \frac{8}{12} = \frac{17}{12} = 1\frac{5}{12}$; $15 + 1\frac{5}{12} = 16\frac{5}{12}$. Ans.

20. A had $25\frac{2}{5}$ acres of land, and then bought $17\frac{3}{4}$ acres; how many had he then?

21. B had $57\frac{3}{5}$ dollars, and C had $96\frac{5}{6}$ dollars; what was the sum of their money?

22. C sold $96\frac{7}{8}$ yards of cloth, and then had $147\frac{9}{16}$ yards left; how much had he at first?

SUBTRACTION OF FRACTIONS.

111. ***Subtraction of Fractions*** is the process of finding the difference between two fractions.

CASE I.

To subtract when the denominators are alike.

1. Subtract $\frac{3}{8}$ from $\frac{7}{8}$.

SOLUTION.—3 *eighths* subtracted from 7 *eighths* leave 4 *eighths*, and $\frac{4}{8}$ reduced to its lowest terms equals $\frac{1}{2}$.

OPERATION.

$\frac{7}{8} - \frac{3}{8} = \frac{4}{8}$, or $\frac{1}{2}$

2. Subtract $\frac{3}{7}$ from $\frac{5}{7}$.
3. Subtract $\frac{1}{4}$ from $\frac{3}{4}$.
4. Subtract $\frac{2}{9}$ from $\frac{8}{9}$.
5. Subtract $\frac{5}{12}$ from $\frac{11}{12}$.
6. Mary had $\frac{7}{8}$ of an apple, and gave away $\frac{3}{8}$ of an apple; what part of an apple had she left? Ans. $\frac{1}{2}$.
7. Peter found $\frac{4}{5}$ of a dollar, and spent $\frac{2}{5}$ of a dollar; what part of a dollar had he left?
8. If I buy $\frac{11}{12}$ of a ton of hay and sell $\frac{5}{12}$ of a ton, what part of a ton will I have left?
9. Peter has $\frac{5}{10}$ of a dollar, John $\frac{7}{10}$ of a dollar, and

Jacob $\frac{7}{10}$ of a dollar; how much more have Peter and John than Jacob? Ans. $\$\frac{1}{2}$.

10. Mary has $\frac{3}{5}$ of a dollar, Sarah $\frac{4}{5}$ of a dollar, and Jane $\frac{2}{5}$ of a dollar; how much more have Mary and Sarah than Jane? Ans. \$1.

CASE II.

To subtract when the denominators are unlike.

1. Subtract $\frac{2}{3}$ from $\frac{4}{5}$.

SOLUTION.—We will first reduce the fractions to a common denominator. $\frac{4}{5}=\frac{12}{15}$ and $\frac{2}{3}=\frac{10}{15}$, and 12 *fifteenths* minus 10 *fifteenths* is 2 *fifteenths*. From this solution we have the following

OPERATION.

$\frac{4}{5}-\frac{2}{3}=$
$\frac{12}{15}-\frac{10}{15}=\frac{2}{15}$

RULE.—*Reduce the fractions to a common denominator, subtract the numerators, and place the result over the common denominator.*

NOTE.—Reduce each fraction, and also the difference, to its lowest terms.

Subtract		Subtract	
2. $\frac{2}{3}$ from $\frac{3}{4}$.	Ans. $\frac{1}{12}$.	8. $\frac{5}{8}$ from $\frac{10}{12}$.	Ans. $\frac{5}{24}$.
3. $\frac{3}{4}$ from $\frac{5}{6}$.	Ans. $\frac{1}{12}$.	9. $\frac{6}{7}$ from $\frac{7}{8}$.	Ans. $\frac{1}{56}$.
4. $\frac{3}{5}$ from $\frac{5}{6}$.		10. $\frac{8}{9}$ from $\frac{13}{14}$.	
5. $\frac{3}{7}$ from $\frac{4}{5}$.		11. $\frac{7}{8}$ from $\frac{11}{12}$.	
6. $\frac{3}{9}$ from $\frac{5}{8}$.		12. $\frac{8}{12}$ from $\frac{9}{10}$.	
7. $\frac{6}{7}$ from $\frac{11}{12}$.		13. $\frac{11}{12}$ from $\frac{14}{15}$.	

14. Mary had $\frac{3}{5}$ of a dollar and gave away $\frac{1}{4}$ of a dollar; how much had she left?

SOLUTION.—If Mary had $\frac{3}{5}$ of a dollar, and gave away $\frac{1}{4}$ of a dollar, she had left the difference between $\frac{3}{5}$ of a dollar and $\frac{1}{4}$ of a dollar, which by reducing to a common denominator and subtracting, we find to be $\frac{7}{20}$ of a dollar

OPERATION.

$\frac{3}{5}-\frac{1}{4}=$
$\frac{12}{20}-\frac{5}{20}=\frac{7}{20}$ Ans.

15. Willie gave Sallie $\frac{4}{5}$ of a quart of peanuts, and Sallie gave him back $\frac{3}{4}$ of a quart; what part of a quart did Sallie keep?

16 A has $\frac{7}{8}$ of a pie; if he gives B $\frac{2}{5}$ of a pie, how much will remain?

17. B bought $\frac{9}{10}$ of a ton of hay, and sold C $\frac{3}{7}$ of a ton; how much did B retain?

18. C owned $\frac{1}{4}$ of a vessel, H bought $\frac{1}{5}$ of this, and then sold $\frac{1}{3}$ of what he bought: how much did H keep?

19. The sum of two fractions is $\frac{19}{20}$, and one fraction is $\frac{11}{12}$; what is the other fraction?

20. If D had $\frac{2}{5}$ of a certain sum of money, and then earned $\frac{3}{4}$ of the same sum, and then spent $\frac{4}{5}$ of the sum, how much remained?

21. The sum of three fractions is $\frac{15}{16}$, and two of these fractions are $\frac{1}{8}$ and $\frac{1}{4}$; what is the third fraction?

22. A has $\frac{7}{8}$ of a sum of money; he owes B $\frac{3}{8}$, and C $\frac{2}{5}$ of that sum of money; how much will A have after paying his debts? Ans. $\frac{1}{10}$.

PRACTICAL PROBLEMS

in Addition and Subtraction of Fractions.

1. Subtract $4\frac{3}{4}$ from $7\frac{1}{4}$.

Solution.—$7\frac{1}{4}$ equals $6 + \frac{4}{4} + \frac{1}{4} = 6\frac{5}{4}$; $4\frac{3}{4}$ subtracted from $6\frac{5}{4}$ leaves $2\frac{2}{4}$.

OPERATION.

$$\begin{array}{r} 7\frac{1}{4} = 6\frac{5}{4} \\ 4\frac{3}{4} \\ \hline 2\frac{2}{4} \end{array}$$ Ans.

Subtract

2. $5\frac{2}{3}$ from $9\frac{1}{3}$.
3. $7\frac{3}{7}$ from $10\frac{2}{7}$.
4. $8\frac{5}{9}$ from $13\frac{2}{9}$.

Subtract

5. $16\frac{2}{3}$ from $20\frac{1}{4}$
6. $19\frac{6}{7}$ from $30\frac{2}{3}$.
7. $24\frac{3}{4}$ from $36\frac{3}{5}$.

8. A had $17\frac{3}{5}$ dollars, and gave B $12\frac{4}{5}$; how much remained?

9. A has $6\frac{3}{4}$ dollars, and B has $7\frac{1}{2}$ dollars; how much have both?

10. A read $4\frac{1}{2}$ pages one day and $7\frac{1}{3}$ another day; how many pages did he read in the two days?

11. Mary had 20 dollars; she gave her brother $\$9\frac{3}{4}$ and her sister $\$7\frac{2}{5}$; how much remained?

12. William having \$100 gave $\$26\frac{2}{3}$ to the poor and spent $\$18\frac{5}{6}$ for clothing; how much remained?

13. My father gave me $\$3\frac{3}{5}$, my mother gave me $\$5\frac{3}{4}$, and I then gave my sister $\$4\frac{1}{4}$; how much remained?

14. What is the sum of $\frac{1}{2}$ of $\frac{1}{3}$ and $\frac{1}{3}$ of $\frac{1}{4}$; and what, also, is their difference?

15. I had \$24, and gave $\frac{1}{3}$ of it to my sister and $\frac{1}{4}$ of it to my brother; how much remained?

16. Sarah had \$23, and gave $\frac{1}{4}$ of it to the poor and $\frac{2}{5}$ of it for a dress; how much remained?

17. Henry's father gave him $\$16\frac{3}{4}$, and his mother gave him $\$18\frac{1}{2}$; he then spent $\frac{2}{3}$ of it; how much remained?

18. A man had $\$24\frac{3}{4}$, and then earned $\$16\frac{1}{5}$, and then spent $\frac{1}{2}$ of it; how much remained?

19. Peter had $\$17\frac{1}{2}$, and then lost $\$11\frac{1}{3}$, and then earned \$14; how much had he then?

20. Harold had $\$\frac{4}{5}$, then lost $\$\frac{3}{4}$, and then earned $\$\frac{5}{8}$; how much had he then?

MULTIPLICATION OF FRACTIONS.

112. ***Multiplication of Fractions*** is the process of multiplying when one or both terms are fractions.

CASE I.

113. To multiply a fraction by a whole number.

1. Multiply $\frac{5}{8}$ by 4.

SOLUTION.—4 times $\frac{5}{8}$ equal $\frac{20}{8}$, according to PRIN. I. Or, 4 times $\frac{5}{8}$ equal $\frac{5}{2}$, since dividing the denominator multiplies the fraction, according to PRIN. IV. From this we have the following	OPERATION. $\frac{5}{8} \times 4 = \frac{20}{8}$ or, $\frac{5}{8} \times 4 = \frac{5}{2}$

RULE.—*To multiply a fraction by an integer, multiply the numerator or divide the denominator by the integer.*

Multiply		Multiply	
2. $\frac{9}{10}$ by 5.	Ans. $\frac{9}{2}$.	6. $\frac{17}{18}$ by 9.	Ans. $8\frac{1}{2}$.
3. $\frac{11}{16}$ by 4.	Ans. $\frac{11}{4}$.	7. $\frac{20}{21}$ by 7.	Ans. $6\frac{2}{3}$.
4. $\frac{14}{15}$ by 7.		8. $\frac{25}{72}$ by 12.	
5. $\frac{18}{19}$ by 3.		9. $\frac{121}{144}$ by 36.	

10. A has $\frac{17}{18}$ of a ton of hay, and B has 3 times as much; how much have both?

CASE II.

114. To multiply a whole number by a fraction.

1 Multiply 8 by $\frac{2}{3}$; also by $4\frac{2}{3}$.

SOLUTION 1.—8 multiplied by $\frac{1}{3}$ equals $\frac{1}{3}$ of 8, or $\frac{8}{3}$, and 8 multiplied by $\frac{2}{3}$ equals 2 times $\frac{8}{3}$, or $\frac{16}{3}$.

OPERATION.

$$8 \times \frac{2}{3} = \frac{8 \times 2}{3} = \frac{16}{3} = 5\frac{1}{3}$$

SOLUTION 2.—We multiply 8 by 2 and divide by 3, and have $5\frac{1}{3}$; then multiply by 4 and add the product 32 to $5\frac{1}{3}$, making $37\frac{1}{3}$. Hence, in a mixed number we multiply first by the fraction, and then by the integer. From these solutions we have the following

OPERATION.

$$\begin{array}{r} 8 \\ \underline{4\frac{2}{3}} \\ 3)\underline{16} \\ 5\frac{1}{3} \\ \underline{32} \\ 37\frac{1}{3} \end{array}$$

RULE.—*Multiply the whole number by the numerator of the fraction, and divide the product by the denominator.*

Multiply		Multiply	
2. 16 by $\frac{3}{4}$.	Ans. 12.	7. 45 by $\frac{8}{9}$.	Ans. 40.
3. 18 by $\frac{5}{6}$.	Ans. 15.	8. 43 by $\frac{5}{6}$.	Ans. $35\frac{5}{6}$
4. 12 by $\frac{7}{8}$.		9. 28 by $5\frac{4}{7}$.	
5. 20 by $2\frac{9}{10}$.		10. 76 by $4\frac{7}{8}$.	
6. 35 by $4\frac{6}{7}$.		11. 85 by $8\frac{9}{10}$.	

12. A has 18 tons of hay, and B has $4\frac{3}{4}$ times as much plus $3\frac{1}{2}$ tons; how much has B?

CASE III.

115. To multiply a fraction by a fraction.

1. Multiply $\frac{3}{8}$ by $\frac{3}{4}$.

SOLUTION.—$\frac{3}{5}$ multiplied by $\frac{1}{4}$ equals $\frac{1}{4}$ of $\frac{3}{5}$, which is $\frac{3}{20}$, according to PRIN. III.; and $\frac{3}{5}$ multiplied by $\frac{3}{4}$ equals 3 times $\frac{3}{20}$ or $\frac{9}{20}$.

OPERATION.

$$\frac{3}{5} \times \frac{3}{4} = \frac{3 \times 3}{5 \times 4} = \frac{9}{20}.$$

RULE.—*Multiply the numerators together for the numerator, and the denominators together for the denominator of the product.*

NOTE.—Reduce the result to its lowest terms.

What is the product of

2. $\frac{3}{4}$ by $\frac{2}{5}$? Ans. $\frac{3}{10}$.
3. $\frac{7}{8}$ by $\frac{4}{5}$? Ans. $\frac{7}{10}$.
4. $\frac{9}{10}$ by $\frac{5}{6}$?
5. $\frac{11}{12}$ by $\frac{9}{14}$?
6. $\frac{18}{20}$ by $\frac{15}{16}$?
7. $\frac{19}{21}$ by $\frac{7}{8}$? Ans $\frac{19}{24}$.
8. $\frac{20}{21}$ by $\frac{28}{25}$? Ans. $\frac{16}{15}$.
9. $\frac{25}{29}$ by $\frac{17}{35}$? Ans. $\frac{85}{203}$.
10. $\frac{2}{3}$ by $\frac{3}{4}$ of $\frac{5}{6}$?
11. $\frac{7}{8}$ of $\frac{4}{5}$ by $\frac{25}{28}$?

12. A has $\frac{7}{8}$ of a ton of hay, and B has $\frac{6}{7}$ as much plus $2\frac{3}{8}$ tons; how much has B?

DIVISION OF FRACTIONS.

116. *Division of Fractions* is the process of dividing when one or both terms are fractional.

CASE I.

117. To divide when the dividend is a fraction.

1. Divide $\frac{8}{9}$ by 4.

SOLUTION.—$\frac{8}{9}$ divided by 4 equals $\frac{2}{9}$, according to PRIN. I. When the numerator will not contain the divisor, we multiply the denominator, according to PRIN. III.

OPERATION.

$$\frac{8}{9} \div 4 = \frac{2}{9}$$

RULE.—*Divide the numerator, or multiply the denominator, by the divisor.*

Divide

2. $\frac{9}{10}$ by 3. Ans. $\frac{3}{10}$.
3. $\frac{8}{11}$ by 4. Ans. $\frac{2}{11}$.
4. $\frac{12}{13}$ by 6.
5. $\frac{9}{11}$ by 4.
6. $\frac{3}{5}$ by 8.

Divide

7. $\frac{12}{13}$ by 7. Ans. $\frac{12}{91}$.
8. $\frac{16}{17}$ by 5 Ans. $\frac{16}{85}$.
9. $\frac{18}{7}$ by 8.
10. $3\frac{1}{4}$ by 9.
11. $5\frac{2}{5}$ by 12

12. A gave $3\frac{1}{5}$ dollars to 6 little girls; how much did each receive?

Case II.

118. To divide when the divisor is a fraction.

1. Divide $\frac{3}{4}$ by $\frac{4}{5}$.

Solution.—$\frac{3}{4}$ divided by 1 equals $\frac{3}{4}$, hence $\frac{3}{4}$ divided by $\frac{1}{5}$ equals 5 times $\frac{3}{4}$, and $\frac{3}{4}$ divided by $\frac{4}{5}$ equal $\frac{1}{4}$ of 5 times $\frac{3}{4}$, or $\frac{5}{4}$ times $\frac{3}{4}$, which equal $\frac{15}{16}$. Hence, we see the divisor becomes inverted, and we have the following

OPERATION.

$$\frac{3}{4} \div \frac{4}{5} =$$
$$\frac{3}{4} \times \frac{5}{4} = \frac{15}{16}$$

Rule.—*Invert the divisor, and multiply the dividend by the resulting fraction.*

Divide		Divide	
2. $\frac{6}{7}$ by $\frac{3}{4}$.	Ans. $\frac{8}{7}$.	8. $\frac{15}{16}$ by $\frac{9}{12}$.	Ans. $1\frac{1}{4}$.
3. $\frac{4}{5}$ by $\frac{2}{3}$.	Ans. $\frac{6}{5}$.	9. $\frac{21}{32}$ by $\frac{14}{18}$.	Ans. $\frac{27}{32}$.
4. $\frac{7}{8}$ by $\frac{5}{6}$.	Ans. $\frac{21}{20}$.	10. $\frac{20}{21}$ by $\frac{18}{35}$.	Ans. $1\frac{23}{27}$.
5. $\frac{9}{10}$ by $\frac{3}{8}$.		11. $\frac{16}{17}$ by $\frac{8}{21}$.	Ans. $2\frac{8}{17}$.
6. $\frac{11}{12}$ by $\frac{8}{9}$.		12. $\frac{12}{15}$ by $\frac{16}{35}$.	
7. $\frac{10}{11}$ by $\frac{11}{10}$.		13. $\frac{32}{36}$ by $\frac{48}{50}$.	

14. How much cloth will $\$4\frac{1}{2}$ buy, at $\$\frac{3}{4}$ per yard?

REDUCTION OF COMPLEX FRACTIONS.

118½. A ***Complex Fraction*** is one whose numerator or denominator or both are fractional.

1. Reduce $\dfrac{\frac{2}{3}}{\frac{4}{5}}$ to a simple fraction.

Solution.—This fraction means that $\frac{2}{3}$ is to be divided by $\frac{4}{5}$, and inverting the divisor and multiplying we have $\frac{2}{3} \times \frac{5}{4}$ which equals $\frac{5}{6}$.

OPERATION.

$$\frac{\frac{2}{3}}{\frac{4}{5}} = \frac{2}{3} \div \frac{4}{5} = \frac{2}{3} \times \frac{5}{4} = \frac{5}{6}$$

Rule.—*Multiply the numerator of the complex fraction by its denominator inverted.*

Or, *Reduce mixed numbers, if any, to fractions; then multiply the numerator of the upper fraction by the denominator of the lower fraction, and the denominator of the upper fraction by the numerator of the lower.*

2. Reduce $\frac{\frac{3}{4}}{\frac{5}{6}}$. Ans. $\frac{9}{10}$.

3. Reduce $\frac{4}{\frac{2}{3}}$. Ans. 6.

4. Reduce $\frac{4}{1\frac{1}{3}}$. Ans. 3.

5. Reduce $\frac{3\frac{1}{3}}{2\frac{1}{2}}$. Ans. $1\frac{1}{3}$.

6. Reduce $\frac{\frac{2}{3} \text{ of } \frac{3}{4}}{3\frac{1}{3}}$. Ans. $\frac{3}{20}$.

7. Reduce $\frac{\frac{2}{3} \text{ of } \frac{4}{5}}{\frac{4}{5} \text{ of } \frac{5}{6}}$. Ans. $\frac{4}{5}$.

8. Reduce $\frac{5\frac{1}{4}}{2 + 1\frac{1}{2}}$. A. $1\frac{1}{2}$.

9. Reduce $\frac{\frac{1}{2} + \frac{1}{3}}{\frac{1}{3} - \frac{1}{4}}$. Ans. 10.

MISCELLANEOUS EXAMPLES.

Reduce to improper fractions.

1. $6\frac{7}{8}$. Ans. $\frac{55}{8}$.
2. $9\frac{5}{7}$. Ans. $\frac{68}{7}$.
3. $12\frac{7}{9}$. Ans. $\frac{115}{9}$.
4. $13\frac{13}{14}$. Ans. $\frac{195}{14}$.
5. $24\frac{24}{25}$. Ans. $\frac{624}{25}$.
6. $35\frac{17}{69}$. Ans. $\frac{2432}{69}$.
7. $132\frac{53}{85}$. Ans. $\frac{11273}{85}$.
8. $345\frac{69}{113}$. Ans. $\frac{39054}{113}$.
9. $547\frac{147}{1268}$. Ans. $\frac{693743}{1268}$.
10. $777\frac{654}{987}$. Ans. $\frac{767553}{987}$.

Reduce to mixed numbers

11. $\frac{146}{12}$. Ans. $12\frac{1}{6}$.
12. $\frac{178}{13}$. Ans. $13\frac{9}{13}$.
13. $\frac{192}{14}$. Ans. $13\frac{5}{7}$.
14. $\frac{157}{48}$. Ans. $3\frac{13}{48}$.
15. $\frac{408}{124}$. Ans. $3\frac{9}{31}$.
16. $\frac{607}{225}$. Ans. $2\frac{157}{225}$.
17. $\frac{1728}{345}$. Ans. $5\frac{1}{115}$.
18. $\frac{20081}{483}$. Ans. $41\frac{278}{483}$.
19. $\frac{830540}{5726}$. Ans. $145\frac{135}{2863}$.
20. $\frac{800009}{60501}$. Ans. $13\frac{1928}{8643}$.

Reduce to lowest terms

21. $\frac{121}{132}$. Ans. $\frac{11}{12}$.
22. $\frac{132}{144}$. Ans. $\frac{11}{12}$.
23. $\frac{144}{180}$. Ans. $\frac{4}{5}$.
24. $\frac{182}{196}$. Ans. $\frac{13}{14}$.
25. $\frac{315}{405}$. Ans. $\frac{7}{9}$.
26. $\frac{543}{840}$. Ans. $\frac{181}{280}$.
27. $\frac{480}{1728}$. Ans. $\frac{5}{18}$.
28. $\frac{2420}{4840}$. Ans. $\frac{1}{2}$.
29. $\frac{1320}{1680}$. Ans. $\frac{11}{14}$
30. $\frac{9072}{12096}$. Ans. $\frac{3}{4}$.

Reduce to simple fractions

31. $\frac{3}{4}$ of $\frac{6}{7}$. Ans. $\frac{9}{14}$.
32. $\frac{5}{8}$ of $\frac{12}{13}$. Ans. $\frac{15}{26}$.
33. $\frac{8}{9}$ of $\frac{15}{16}$. Ans. $\frac{5}{6}$.
34. $\frac{2}{5}$ of $4\frac{1}{6}$. Ans. $1\frac{2}{3}$.
35. $\frac{3}{7}$ of $8\frac{2}{5}$. Ans. $3\frac{3}{5}$.
36. $\frac{5}{8}$ of $8\frac{4}{5}$. Ans. $5\frac{1}{2}$.
37. $\frac{13}{14}$ of $\frac{16}{39}$. Ans. $\frac{8}{21}$.
38. $\frac{15}{16}$ of $\frac{24}{25}$. Ans. $\frac{9}{10}$.
39. $7\frac{17}{18}$ of $\frac{27}{35}$. Ans. $6\frac{9}{70}$.
40. $12\frac{53}{81}$ ot $4\frac{43}{56}$. Ans. $61\frac{1}{7}$

Find the value of

41. $\frac{4}{5} + \frac{5}{6}$. Ans. $1\frac{19}{30}$.
42. $\frac{5}{6} + \frac{7}{8}$. Ans. $1\frac{17}{24}$.
43. $\frac{3}{10} + \frac{11}{12}$. Ans. $1\frac{13}{60}$.
44. $4\frac{3}{4} + 5\frac{4}{5}$. Ans. $10\frac{11}{20}$.
45. $6\frac{5}{6} + 7\frac{7}{8}$. Ans. $14\frac{17}{24}$.
46. $9\frac{3}{4} + 8\frac{7}{10}$. Ans. $18\frac{9}{20}$.
47. $\frac{2}{3} + \frac{3}{4} + \frac{5}{8}$. Ans. $2\frac{1}{24}$.
48. $\frac{4}{7} + \frac{9}{10} + \frac{11}{14}$. A. $2\frac{9}{35}$.
49. $\frac{11}{12} + \frac{14}{15} + \frac{15}{16}$. A. $2\frac{63}{80}$.
50. $\frac{14}{15} + \frac{23}{40} + \frac{77}{88}$. A. $2\frac{23}{60}$.

Find the value of

51. $\frac{8}{9} - \frac{5}{6}$. Ans. $\frac{1}{18}$.
52. $\frac{11}{12} - \frac{3}{8}$. Ans. $\frac{13}{24}$.
53. $\frac{13}{14} - \frac{13}{21}$. Ans. $\frac{13}{42}$.
54. $8\frac{2}{3} - 5\frac{3}{4}$. Ans. $2\frac{11}{12}$.
55. $9\frac{3}{5} - 6\frac{3}{8}$. Ans. $3\frac{9}{40}$.
56. $12\frac{7}{9} - 10\frac{5}{6}$. Ans. $1\frac{17}{18}$.
57. $\frac{3}{8} + \frac{5}{9} - \frac{3}{4}$. Ans. $\frac{13}{72}$.
58. $\frac{6}{7} + \frac{5}{8} - \frac{7}{12}$. Ans. $\frac{151}{168}$.
59. $\frac{8}{9} - \frac{7}{8} + \frac{9}{10}$. Ans. $\frac{329}{360}$.
60. $\frac{10}{11} - \frac{5}{13} - \frac{67}{143}$. A. $\frac{8}{143}$.

Find the value of

61. $\frac{3}{4} \times 6$. Ans. $4\frac{1}{2}$.
62. $\frac{5}{6} \times 8$. Ans. $6\frac{2}{3}$.
63. $\frac{7}{8} \times 12$. Ans. $10\frac{1}{2}$.
64. $5\frac{3}{4} \times 8$. Ans. 46.
65. $7\frac{8}{9} \times 12$. Ans. $94\frac{2}{3}$.
66. $8\frac{5}{9} \times 15$. Ans. $128\frac{1}{3}$.
67. $\frac{5}{7} \times \frac{6}{8} \times \frac{14}{15}$. Ans. $\frac{1}{2}$.
68. $\frac{11}{12} \times \frac{15}{16} \times \frac{18}{22}$. A. $\frac{45}{64}$.
69. $\frac{16}{21} \times \frac{18}{25} \times \frac{35}{36}$. A. $\frac{8}{15}$.
70. $\frac{27}{28} \times \frac{49}{54} \times \frac{64}{91}$. A. $\frac{8}{13}$.

Find the value of

71. $\frac{6}{7} \div 4$. Ans. $\frac{3}{14}$.
72. $\frac{8}{9} \div 6$. Ans. $\frac{4}{27}$.
73. $\frac{12}{13} \div 8$. Ans. $\frac{3}{26}$.
74. $4 \div \frac{6}{7}$. Ans. $4\frac{2}{3}$.
75. $16 \div \frac{8}{9}$. Ans. 18.
76. $5\frac{1}{3} \div 12$. Ans. $\frac{4}{9}$.
77. $\frac{11}{12} \div 5\frac{1}{2}$. Ans. $\frac{1}{6}$.
78. $\frac{14}{15} \div 3\frac{1}{2}$. Ans. $\frac{4}{15}$.
79. $6\frac{7}{8} \div 8\frac{1}{4}$. Ans. $\frac{5}{6}$.
80. $9\frac{5}{7} \div 12\frac{3}{14}$. Ans. $\frac{136}{171}$.

Find the value of

81. $\dfrac{\frac{3}{4}}{3\frac{3}{4}}$. Ans. $\frac{1}{5}$.
82. $\dfrac{\frac{4}{5}}{4\frac{4}{5}}$. Ans. $\frac{1}{6}$.
83. $\dfrac{\frac{5}{6}}{6\frac{1}{4}}$. Ans. $\frac{2}{15}$.
84. $\dfrac{8\frac{3}{4}}{9\frac{1}{6}}$. Ans. $\frac{21}{22}$.
85. $\dfrac{\frac{1}{2} + \frac{1}{3}}{\frac{1}{3} + \frac{1}{4}}$. Ans. $1\frac{3}{7}$.
86. $\dfrac{\frac{1}{3} - \frac{1}{5}}{\frac{1}{4} - \frac{1}{6}}$. Ans. $1\frac{3}{5}$.
87. $\dfrac{\frac{2}{3}}{\frac{4}{5}} + \dfrac{1\frac{1}{2}}{2\frac{1}{4}}$. Ans. $1\frac{1}{2}$.
88. $\dfrac{5\frac{1}{3}}{2\frac{2}{5}} + \dfrac{6\frac{1}{4}}{4\frac{3}{8}}$. Ans. $3\frac{41}{63}$.
89. $\dfrac{4\frac{3}{4}}{7\frac{2}{5}} \div \dfrac{6\frac{1}{3}}{5\frac{2}{7}}$. Ans. $\frac{15}{28}$.
90. $\dfrac{6\frac{2}{3}}{9\frac{5}{6}} \times \dfrac{7\frac{3}{4}}{8\frac{4}{5}}$. Ans. $\frac{775}{1298}$.

PRACTICAL PROBLEMS.

1. What cost 24 apples at $\frac{3}{4}$ of a cent each?
Ans. 18 cents.

2. What cost 45 oranges at $2\frac{2}{5}$ cents a piece?
Ans. \$1.08.

3. What cost $16\frac{2}{3}$ yards of muslin at $12\frac{1}{2}$ cents a yard?
Ans. \$2.08$\frac{1}{3}$.

4. What cost 6 quarts of berries at $18\frac{3}{4}$ cents a quart?
Ans. \$1.12$\frac{1}{2}$.

5. What cost 8 yards of ribbon at $12\frac{1}{2}$ cents a yard?
Ans. \$1.00.

6. What cost 4 bushels of oats at $62\frac{1}{2}$ cents a bushel?
Ans. \$2.50.

7. What cost 3 quarts of nuts at $8\frac{1}{3}$ cents a quart?
Ans. 25 cents.

8. What cost 12 bushels of apples at \$$\frac{3}{4}$ a bushel?
Ans. \$9.

9. What cost $6\frac{3}{4}$ yards of cloth at \$$2\frac{2}{3}$ a yard?
Ans. \$18.

10. What cost $13\frac{1}{2}$ pounds of fish at $9\frac{3}{4}$ cents a pound?
Ans. \$1.31$\frac{5}{8}$.

11. What cost $18\frac{3}{4}$ yards of ribbon at $31\frac{1}{4}$ cents a yard?
Ans. \$5.85$\frac{15}{16}$.

12. What cost $26\frac{3}{8}$ pounds of raisins, at $18\frac{3}{4}$ cents a pound?
Ans. \$4.94$\frac{17}{32}$.

13. If one yard of cloth cost \$8, how many yards can be bought for \$47?
Ans. $5\frac{7}{8}$ yards.

14. A boy had \$$5\frac{3}{4}$ and found \$$6\frac{2}{5}$; how much money had he then?
Ans. \$12.15.

15. A man had $9\frac{1}{2}$ tons of hay and sold $4\frac{3}{4}$ tons of it; how much had he left?
Ans. $4\frac{3}{4}$ tons.

16. A had $27\frac{2}{3}$ acres of land and bought $21\frac{1}{4}$ acres; how much land had he then?
Ans. $48\frac{11}{12}$ acres.

17. The sum of two fractions is $\frac{17}{18}$ and one fraction is $\frac{4}{9}$; what is the other fraction?
Ans. $\frac{1}{2}$

18. A man earned $\$25\frac{3}{4}$ and spent $\$18\frac{1}{5}$; how much money remained? Ans. $7.55.

19. Peter had $\$26\frac{1}{2}$ and gave $\$12\frac{3}{4}$ to his sister; how much did he keep? Ans. $13.75.

20. A. sold 5 bushels more than $\frac{1}{3}$ of 40 bushels of apples; how many bushels remained? Ans. $21\frac{2}{3}$.

21. Mary had $25 and spent $\frac{1}{2}$ of it for a dress, and $\frac{1}{2}$ of the remainder for a bonnet; how much then remained? Ans. $6.25.

22. A boy earned $\$18\frac{3}{4}$ and then had $\$45\frac{1}{2}$; how much had he at first? Ans. $26.75.

23. How many bushels of potatoes can be bought for $15 at $\$\frac{3}{4}$ a bushel? Ans. 20 bushels.

24. How many pounds of tea, at $\$1\frac{1}{8}$ a pound can be bought for $\$40\frac{1}{2}$? Ans. 36 lb.

25. A man bought $12\frac{1}{2}$ yards of cloth for $\$62\frac{1}{2}$; what did he pay a yard? Ans. $5.

26. How many tons of coal, at $\$6\frac{3}{4}$ a ton, can be bought for $72? Ans. $10\frac{2}{3}$ tons.

27. If $8\frac{3}{4}$ pounds of grapes cost 49 cents, how much is that a pound? Ans. $5\frac{3}{5}$ cents.

28. How many yards of cloth, at $\$5\frac{5}{8}$ a yard, can be bought for $\$18\frac{3}{4}$? Ans. $3\frac{1}{3}$ yards.

29. How many yards of tape, at $6\frac{1}{4}$ cents a yard, can be bought for $58\frac{3}{4}$ cents? Ans. $9\frac{2}{5}$ yards.

30. How many bushels of wheat, at $\$1\frac{3}{5}$ dollars a bushel, can be bought for $\$242\frac{2}{5}$? Ans. $151\frac{1}{2}$ bushels.

31. How many sheep, at $\$8\frac{3}{4}$ a head, can be bought for $\$157\frac{1}{2}$? Ans. 18.

32. A lady bought $25\frac{1}{2}$ yards of muslin for $\$6.24\frac{3}{4}$; what was the price per yard? Ans. $24\frac{1}{2}$ cents.

33. How much land can be bought for $\$543\frac{1}{4}$, at $\$43\frac{1}{2}$ an acre? Ans. $12\frac{85}{174}$ acres.

34. A servant girl bought $15\frac{1}{8}$ pounds of meat for $\$2.18\frac{3}{4}$; what was the price a pound? A. $14\frac{56}{121}$ cents.

35. A man paid $1566 for cows, giving $65¼ a head; how many did he buy? Ans. 24.

36. How many yards of muslin, at 16⅔ cents a yard, can you buy for $2.08⅓? Ans. 12½ yards.

37. The product of two fractions is $\frac{3}{8}$, and one fraction is $\frac{14}{15}$; what is the other fraction? Ans. $\frac{45}{112}$.

38. What will 3571 feet of lumber cost at $30⅝ per thousand? Ans. 109.36\frac{3}{16}$.

39. The quotient of two numbers is $\frac{19}{20}$, and the divisor is $\frac{9\frac{1}{2}}{20}$; what is the dividend? Ans. $\frac{361}{800}$.

40. If the receipts of the Pennsylvania Railroad for one year are $3,542,000, and the expenses are $\frac{60\frac{37}{100}}{100}$ of the receipts, what are the expenses? A. $2,138,305.40.

ARITHMETICAL ANALYSIS.

119. ***Analysis*** is the process of solving problems by a comparison of their elements. In comparing, we reason to the unit and from the unit, the unit being the basis of the reasoning process.

Case I.

120. To pass from one integer to another.

1. If 5 cows cost $80, what will 7 cows cost at the same rate?

SOLUTION.—If 5 cows cost $80, *one* cow costs $\frac{1}{5}$ of $80, which is $16, and 7 cows will cost 7 times $16, which are $112.

OPERATION.

$$\begin{array}{r} 5)\overline{80} \\ 16 \\ \underline{7} \\ 112 \text{ Ans.} \end{array}$$

2. If 6 hens cost 186 cents, what will 9 hens cost at the same rate?

3. If 5 pigs cost $35, what will 11 pigs cost at the same rate?

4. If 8 horses cost $1200, what will 12 horses cost at the same rate?

5. If 7 yards of cloth cost $42, what will 25 yards cost at the same rate?

6. How much must I pay for 36 cows, at the rate of 7 cows for 196 dollars?

7. What will 17 books cost, at the rate of 8 books fo $10.80?

8. A man bought 72 ducks at the rate of $21 for 7; what did they cost?

9. If a man can walk 324 miles in 9 days, how far can he walk in 69 days?

10. In 26 years there are 9490 days; how many days are there in 75 years?

11. In 5 square miles there are 3200 acres; how many acres in 64 square miles?

12. If a car run 2736 miles in 18 days, how far will it run in 54 days?

CASE II.

121. To pass from a fraction to an integer.

1. If $\frac{2}{3}$ of an acre of land cost $96, what will one acre cost?

OPERATION

$\frac{2}{3} = \$96$

$\frac{1}{3} = \$48$

$\frac{3}{3} = \$144$ Ans.

SOLUTION.—If $\frac{2}{3}$ of an acre cost $96, $\frac{1}{3}$ of an acre cost $\frac{1}{2}$ of $96, or $48, and if $\frac{1}{3}$ of an acre cost $48, $\frac{3}{3}$ of an acre, or one acre, will cost 3 times $48, or $144.

2. If $\frac{3}{4}$ of a sum of money is $72, required the sum.

3. If $\frac{5}{6}$ of the cost of a cow is $25, required the cost of the cow.

4. What cost 2 boxes of raisins, if $\frac{3}{5}$ of a box cost 6 dollars?

5. What is the distance from Lancaster to Philadelphia, if $\frac{3}{4}$ of the distance is 51 miles?

6 If the cost of $\frac{4}{6}$ of an acre of land is $120, what will 4 acres cost at the same rate?

7. If $\frac{6}{7}$ of a farm cost $7200, what will the whole farm cost at that rate?

8. How much will 7 loads of hay weigh, if $\frac{7}{9}$ of a load weighs 840 pounds?

9. What will 17 horses cost me, if $\frac{3}{7}$ of the price of a horse is 93 dollars?

10. A merchant bought 236 barrels of flour at the rate of $8 for $\frac{4}{5}$ of a barrel; how much did they cost him?

CASE III.

122. To pass from a unit or fraction to a fraction.

1. If one barrel of flour costs $12, what will $\frac{3}{4}$ of a barrel cost?

SOLUTION.—If one barrel of flour costs $12, 1 fourth of a barrel will cost $\frac{1}{4}$ of $12, or $3, and $\frac{3}{4}$ of a barrel will cost 3 times $3, or $9.

OPERATION.

$$\begin{array}{r} 4)12 \\ \hline 3 \\ 3 \\ \hline 9 \end{array} \text{ Ans.}$$

2. If one acre of land is worth $125, what is $\frac{4}{5}$ of an acre worth?

3. A paid $1650 for a pleasure-boat; how much would he have paid if he had given $\frac{5}{6}$ as much?

4. If $\frac{2}{3}$ of a barrel of flour cost $8, what will $\frac{3}{4}$ of a barrel cost?

5. If there are 40 pounds in $\frac{2}{3}$ of a bushel of clover-seed, how many pounds are there in $\frac{5}{6}$ of a bushel?

6. If there are 50 pounds in $\frac{5}{6}$ of a bushel of wheat, how many pounds are there in $\frac{11}{12}$ of a bushel?

7. If there are 49 pounds in $\frac{7}{8}$ of a bushel of rye, how many pounds are there in $\frac{5}{7}$ of a bushel?

8. If there are 147 pounds in $\frac{3}{4}$ of a barrel of flour, how many pounds are there in $\frac{6}{7}$ of a barrel?

9. If there are 154 cubic inches in $\frac{2}{3}$ of a gallon, how many cubic inches in $\frac{5}{7}$ of a gallon?

10. If there are 1536 cubic inches in $\frac{8}{9}$ of a cubic foot, how many cubic inches in $\frac{11}{12}$ of a cubic foot?

Case IV.

123. Given a fractional part and the remainder, to find the whole.

1 A man spent $\frac{3}{5}$ of his money, and then had $24 remaining; how much money had he at first?

Solution.—If he spent $\frac{3}{5}$ of his money, there remained $\frac{5}{5}$ of his money minus $\frac{3}{5}$ of his money, which is $\frac{2}{5}$ of his money, which is $24. If $\frac{2}{5}$ of his money is $24, $\frac{1}{5}$ of his money is $\frac{1}{2}$ of $24, which is $12. and $\frac{5}{5}$ of his money is 5 times $12, or $60.

OPERATION.

$$\frac{5}{5} - \frac{3}{5} = \frac{2}{5} = \$24$$
$$\frac{1}{5} = \$12$$
$$\frac{5}{5} = \$60 \text{ Ans.}$$

2. A man spent $\frac{3}{5}$ of his money, and then had $30 remaining; how much had he at first?

3. William sold $\frac{2}{5}$ of his hens, and then had 60 remaining; how many had he at first?

4. Henry sold $\frac{2}{7}$ of his bank-stock, and the remainder was worth $550; how much had he at first?

5. After giving $\frac{1}{4}$ of his income to the poor, Samuel had $960 remaining; what was his income?

6. A pole stands $\frac{1}{3}$ in the mud and $\frac{1}{4}$ in the water, and 12 feet in the air; required the length of the pole.

7. One-fourth of a drove of animals are cows, $\frac{1}{5}$ are pigs, and the remainder, 132, are sheep; how many animals in the drove?

8. Two-fifths of my money is in bank, $\frac{1}{3}$ in government bonds, and $480 in cash; what was my money?

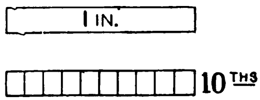
9. A sold $\frac{1}{4}$ of his land to B, and $\frac{3}{7}$ to C, and then had 90 acres remaining; how much had he at first?

10. A man walked $\frac{2}{7}$ of the distance from Lancaster to Philadelphia one day, $\frac{2}{5}$ of the distance the next day, and the remaining distance, 22 miles, the third day; how far did he walk each day? Ans. 20; 28; 22.

INTRODUCTION TO DECIMALS.

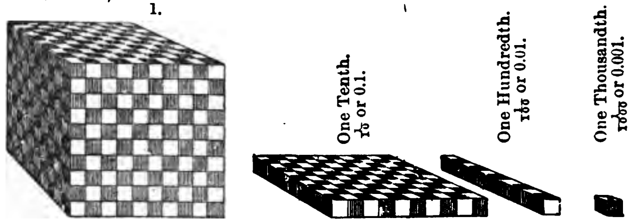
IDEAS OF DECIMALS.

1. If an inch is divided into ten equal parts, what is one of these parts called?



2. How many tenths in one inch? How many tenths in one half an inch?

A Block, or One.
1.



3. If a block is divided into 10 equal parts, what is one of the parts called?

4. If one-tenth of the block is divided into 10 equal parts, what is each of these parts called?

5. When, then, is $\frac{1}{10}$ of $\frac{1}{10}$? How many hundredths in $\frac{1}{10}$? In 1?

6. If 1 hundredth of the block is divided into 10 equal parts, what is each part called?

7. What is $\frac{1}{10}$ of $\frac{1}{100}$? How many thousandths in $\frac{1}{100}$? In $\frac{1}{10}$? In 1?

9. These fractions produced by the successive division by *ten* are called *decimal fractions.* The word *decimal* is derived from *decem,* meaning *ten.*

WRITING DECIMALS.

1. How do we write one-tenth? 2 tenths? 3 tenths? 1 hundredth? 2 hundredths? 1 thousandth? 5 thousandths?

2. There is another, simpler way of writing decimal fractions similar to that used for whole numbers which we will now try to learn.

3. In the number 333 the 3 *units* is what part of the 3 *tens?* Ans. *One-tenth.*

4. The 3 *tens* is what part of the 3 *hundreds?* Ans. *One-tenth.*

5. A figure, then, in units' place denotes what part of the value it does in tens' place? Ans. *One-tenth.*

6. We thus see that a figure in any place denotes $\frac{1}{10}$ of what it does in the next place to the left.

7. Suppose that we place a 3 to the *right* of 333, using a period to separate it from units' place—thus: 333.3; what part of a unit will this 3 denote? Ans. 3 *tenths.*

8. In this way how would we express $2\frac{5}{10}$? Ans. 2.5.

9. How would we express $8\frac{7}{10}$? $12\frac{4}{10}$? $19\frac{6}{10}$? $24\frac{9}{10}$?

10. In the expression 45.23 the figure 2 expresses only a tenth as much as if it were in units' place, and hence expresses 2 *tenths.*

11. The figure 3 denotes only a tenth as much as if it were in tenths' place, hence it expresses 3 *hundredths.*

12. What shall we call the first place to the right of units? Ans. *Tenths.* The second place to the right? Ans. *Hundredths.* The third place? Ans. *Thousandths.*

13. Write in this way 4 and 2 tenths; 4 and 5 tenths; 4 and 7 tenths; 4 and 3 hundredths; 4 and 6 hundredths; 4 and 8 hundredths.

14. Read the following expressions: 2.3; 7.5; 14.05; 16.25; 32.75; 25.006; 35.205.

15. Read the following expressions: .5; .8; .06; .07; .25; .36; .005; .008; .035; .245.

16. Write the following as decimals: $\frac{8}{10}$; $\frac{5}{10}$; $\frac{25}{100}$; $\frac{46}{100}$; $\frac{7}{100}$; $\frac{8}{100}$; $\frac{125}{1000}$; $\frac{487}{1000}$; $\frac{23}{1000}$; $\frac{9}{1000}$.

NOTE.—In this way we can readily lead the pupil to see how a decimal fraction is written, and the reason for its being thus written.

Another way of teaching pupils to write decimals is not to show the reason for it, but merely to tell them that $\frac{2}{10}$ is written thus, .2; that $\frac{5}{100}$ is written .05, etc. But, taught in this way, they will see no reason why it should be thus written.

SECTION VI.

DECIMAL FRACTIONS.

124. A ***Decimal Fraction*** is a number of the decimal divisions of a unit; that is, a number of *tenths, hundredths,* etc.

125. A decimal fraction is usually expressed by placing a point before the numerator and omitting the denominator. Thus, .5 represents $\frac{5}{10}$; .05 represents $\frac{5}{100}$, etc.

126. The point is called the *decimal point,* or *separatrix.* The decimal fraction thus expressed is called a *decimal.*

127. This method of expressing decimal fractions is but an extension of the method of notation for integers. This method, as applied to integers and fractions, is exhibited in the following

NOTATION AND NUMERATION TABLE.

	Ten-millions.	Millions.	Hundred-thousands.	Ten-thousands.	Thousands.	Hundreds.	Tens.	Units.	Tenths.	Hundredths.	Thousandths.	Ten-thousandths.	Hundred-thousandths.	Millionths.	Ten-millionths.
	6	6	6	6	6	6	6	6.	6	6	6	6	6	6	6
PLACES.	8th,	7th,	6th,	5th,	4th,	3d,	2d,	1st,	2d,	3d,	4th,	5th,	6th,	7th,	8th,

EXAMPLES IN NUMERATION.

1 Read the decimal .36.

SOLUTION.—This expresses 3 tenths and 6 hundredths, or, since 3 tenths equals 30 hundredths, and 30 hundredths plus 6 hundredths equal 36 hundredths, it may also be read 36 hundredths.

128. Hence there are two methods of reading decimals, which are expressed by the following rules:—

RULE 1.—*Begin at tenths, and read the terms in order toward the right, giving each its proper denomination.*

RULE 2.—*Read the decimal as a whole number, and give it the denomination of the last term on the right; numerating toward the point to determine the numerator, and from the point to determine the denominator.*

Read the following decimals:—

2. .45.	6. .046.	10. 2.0123.
3. .83.	7. .007.	11. 4.2057.
4. .126.	8. .3216.	12. 13.0205.
5. .324.	9. .1357.	13. 27.0027.

EXAMPLES IN NOTATION.

1. Express 25 hundredths in the form of a decimal.

SOLUTION.—25 hundredths equals 2 tenths and 5 hundredths, and this is expressed by writing a decimal point before 25, thus, .25 Hence the following

RULE.—*Write the decimal as we would a whole number, placing the decimal point so as to give each figure its proper place, using ciphers after the decimal point if necessary.*

Express the following in the decimal form.

2. Thirty-four hundredths.
3. Seventy-five hundredths.
4. Two-tenths and six-hundredths.
5. Twenty-five thousandths.
6. Four-tenths and 7-thousandths.
7 Seven-tenths and 8-thousandths.
8 Five hundred and 25-thousandths.
9. Three-tenths and 7 ten-thousandths.
10. Four hundredths and 96 millionths. Ans. .040096.

PRINCIPLES OF DECIMAL NOTATION.

129. We now present the following principles of Decimals, which the pupils will illustrate.

1. *Changing the decimal point one place toward the right multiplies by* 10; *two places, by* 100, etc.

2. *Changing the decimal point one place toward the left divides by* 10; *two places, by* 100, etc.

3. *Placing a cipher between the decimal point and a decimal divides the decimal by ten.*

REDUCTION OF DECIMALS.

130. The ***Reduction of Decimals*** is the process of changing their form without changing their value.

There are two cases:—

1. To reduce decimals to common fractions.
2. To reduce common fractions to decimals.

Case I.

131. To reduce a decimal to a common fraction.

1. Reduce .45 to a common fraction.

Solution.—.45 expressed in the form of a common fraction, is $\frac{45}{100}$, which, reduced to its lowest terms, equals $\frac{9}{20}$. Hence we have the following

OPERATION.

$.45 = \frac{45}{100}$
$= \frac{9}{20}$ Ans.

Rule.—*Write the denominator under the decimal, omitting the decimal point, and reduce the common fraction to its lowest terms.*

Reduce the following decimals to common fractions.

2. .35.	Ans. $\frac{7}{20}$.	6. 9.75.	Ans. $9\frac{3}{4}$.
3. .48.	Ans. $\frac{12}{25}$.	7. .725.	Ans. $\frac{29}{40}$.
4. .125.	Ans. $\frac{1}{8}$.	8. .075.	Ans. $\frac{3}{40}$.
5. .625.	Ans. $\frac{5}{8}$.	9. .0125.	Ans. $\frac{1}{80}$.

Case II.

132. To reduce a common fraction to a decimal.

1. Reduce $\frac{3}{4}$ to a decimal.

SOLUTION.—$\frac{3}{4}$ equals $\frac{1}{4}$ of 3. 3 equals 30 tenths, and $\frac{1}{4}$ of 30 tenths is 7 tenths and 2 tenths remaining. 2 tenths equals 20 hundredths, and $\frac{1}{4}$ of 20 hundredths is 5 hundredths; hence $\frac{3}{4} = .75$. From this we have the following

OPERATION.

$\frac{3}{4} = \frac{1}{4}$ of $3 =$

$$4\overline{)3.00}$$
$$.75$$

RULE.—1. *Annex ciphers to the numerator and divide by the denominator.*

2. *Point off as many places in the quotient as there are ciphers annexed.*

Reduce the following common fractions to decimals.

2. $\frac{1}{4}$.	Ans. .25.	7. $\frac{7}{16}$.	Ans. .4375.
3. $\frac{1}{8}$.	Ans. .125.	8. $\frac{9}{16}$.	Ans. .5625.
4. $\frac{5}{8}$.		9. $\frac{11}{25}$.	
5. $\frac{7}{8}$.		10. $\frac{13}{25}$.	
6. $\frac{5}{16}$.	Ans. .3125.	11. $\frac{15}{64}$.	

ADDITION OF DECIMALS.

133. ***Addition of Decimals*** is the process of finding the sum of two or more decimals.

1. What is the sum of 7.5, 18.25, 21.36 and 47.45?

SOLUTION.—We write the numbers so that figures of the same order shall stand in the same column, and commence at the right to add. 5 hundredths, plus 6 hundredths, plus 5 hundredths, equal 16 hundredths, which equal 1 tenth and 6 hundredths; we write the 6 hundredths, and add the 1 tenth to the next sum. 4 tenths, plus 3 tenths, plus 2 tenths, plus 5 tenths, are 14 tenths, and the 1 tenth added are 15 tenths, which equals 1 unit and 5 tenths; we write the 5 tenths, and add the 1 unit to the sum of the units, etc.

OPERATION.

$$\begin{array}{r} 7.5 \\ 18.25 \\ 21.36 \\ 47.45 \\ \hline 94.56 \end{array}$$

RULE.—1. *Write the numbers so that units of the same order shall stand in the same column.*

2. *Add, as in whole numbers, placing the decimal point in its proper place in the sum.*

2. Find the sum of 12.05, 33.24, 47.62, 96.47.
3. Find the sum of 76.24, 89.45, 36.40, 85.75.
4. Find the sum of 79.76, 85.08, 95.42, 237.675.
5. Add 18.79, 147.072, 856.709, 185.8761, 397.05784.
6. Add 59.874, 435.095, 672.328, 976.309, 8467.500843.
7. Add together 9 and 7 tenths, 41 and 8 hundredths, 75 and 54 hundredths, 128 and 187 thousandths.
Ans. 254.507.
8. Add together 76 and 49 hundredths, 127 and 49 thousandths, 496 and 167 thousandths, 985 and 98 ten-thousandths, and 99 and 99 hundred-thousandths.

SUBTRACTION OF DECIMALS.

134. ***Subtraction of Decimals*** is the process of finding the difference between two decimals.

1. From 67.35 take 42.63.

SOLUTION.—We write the numbers so that figures of the same order stand in the same column, and begin at the right to subtract. 3 hundredths from 5 hundredths leave 2 hundredths; 6 tenths we cannot subtract from 3 tenths; we therefore take 1 unit from the 7 units, which with 3 tenths equal 13 tenths; then 6 tenths from 13 tenths leave 7 tenths, etc.

OPERATION
$$\begin{array}{r} 67.35 \\ \underline{42.63} \\ 24.72 \end{array}$$

RULE.—1. *Write the smaller number under the greater, so that figures of the same order stand in the same column.*

2. *Subtract as in simple numbers, and place the decimal point in its proper place* ***in the difference.***

2. From 63.72 take 25.81.
3. From 96.32 take 73.15.
4. From 123.16 take 75.84.
5. From 247.125 take 167.183.
6. From 1 and 1 tenth take 1 tenth and 1 thousandth.
7. From 2 and 2 hundredths take 2 tenths and 2 thousandths.
8. From 3 tenths take 3 ten-thousandths.
9. From 7 take 7 tenths and 707 millionths.

MULTIPLICATION OF DECIMALS.

135. ***Multiplication of Decimals*** is the process of multiplying when one or both terms are decimals.

1. Multiply 7.23 by .46.

Solution 1.—Multiplying as in whole numbers, we have 33258; now, if the multiplicand alone were hundredths, the product would be one-hundredth of this, or 332.58; but since the multiplier is also hundredths, the product is one-hundredth of 332.58, which, by moving the decimal point two places to the left, becomes 3.3258.

OPERATION.

$$\begin{array}{r} 7.23 \\ .46 \\ \hline 4338 \\ 2892 \\ \hline 3.3258 \end{array}$$

Solution 2.—$7.23 \times .46 = \frac{723}{100} \times \frac{46}{100} = \frac{723 \times 46}{10000} = \frac{1}{10000} \times 33258 = 3.3258$. From either of these solutions we derive the following

Rule.—*Multiply as in whole numbers, and point off as many decimal places in the product as there are in both multiplier and multiplicand, prefixing ciphers when necessary.*

2. Multiply 15.17 by .18.
3. Multiply 26.18 by .25.
4. Multiply 53.46 by .35.
5. Multiply 67.38 by 1.26.
6. Multiply 138.25 by 2.47.
7. Multiply 466.72 by 5.29.
8. Multiply 407.03 by 7.35.
9. Multiply 620.75 by 12.36.
10. Multiply 725.82 by 23.08.
11. Multiply .00723 by .0317.
12. Multiply 1.0309 by .00321.

DIVISION OF DECIMALS.

136. ***Division of Decimals*** is the process of dividing when one or both terms are decimals.

1. Divide 7.8315 by 2.27.

	OPERATION.
SOLUTION.—If we divide as in whole numbers, we obtain a quotient of 345.; now, since the dividend is the product of the divisor and quotient, the number of decimal places in the dividend must equal the number in the divisor and quotient; hence, the number of decimal places in the quotient must equal the number of decimal places in the dividend diminished	2.27)7.8315(3.45 Ans. 6 81 1 021 908 1135 1135

by the number in the divisor; hence, there should be *four* minus *two*, or *two* decimal places in the quotient, therefore the quotient is 3.45.

SOLUTION 2.—$7.8315 \div 2.27 = \frac{78315}{10000} \div \frac{227}{100} = \frac{78315}{10000} \times \frac{100}{227} = \frac{78315}{100 \times 227} = \frac{1}{100} \times \frac{78315}{227} = \frac{1}{100} \times 345 = 3.45$. From either of these solutions we derive the following

RULE.—*Divide as in whole numbers, and point off as many decimal places in the quotient as the number of decimal places in the dividend exceeds the number in the divisor.*

NOTE 1.—When there are not as many decimal places in the dividend as in the divisor, annex ciphers to make the number of places equal.

2. When the number of figures in the quotient is less than the excess of the decimal places in the dividend over those in the divisor, ciphers must be prefixed to the quotient.

2. Divide 14.1372 by 4.5. Ans. 3.1416.
3. Divide 196.1875 by 10.75. Ans. 18.25.
4. Divide 25.1328 by 8. Ans. 3.1416.
5. Divide 65.9736 by 3.1416.
6. Divide 2450.448 by .5236.
7. Divide 2748.9 by .7854.
8. Divide 127.328 by .07958.
9. Divide 15.90435 by 20.25.
10. Divide 352.0625 by 32.75.

PRACTICAL PROBLEMS.

1. What cost 43.45 acres of land at $38.50 an acre? Ans. $1672.825.
2. What cost 57.75 tons of hay at $12.25 a ton? Ans. $707.4375.

3. If 31.25 yards of muslin cost $7 8125; how much is that a yard? Ans. $0.25.

4. A man sold 35.25 pounds of butter for $5.875; how much is that a pound? Ans. $0.166+.

5. There are 7.92 inches in a link; how many inches in 990 links? Ans. 7840.8 in.

6. There are 31.5 gallons in a barrel; how many barrels in 2756.25 gallons? Ans. 87.5 barrels.

7 If 14.5 yards of cloth cost $68.875, how much is that a yard? Ans. $4.75.

8. If a man walk 112·1184 miles in 9.16 days, how many miles does he walk each day? Ans. 12.24 miles.

9. How many yards of cloth at $4.28 a yard, can a person buy for $44.9828? Ans. $10.51.

10. What is the value of 54.6 multiplied by 80.5, and the product divided by 2? Ans. 2197.65.

11. The circumference of a water-wheel is 64 feet, and the diameter equals this divided by 3.1416; required the diameter? Ans. 20.3718 feet.

12. If 25.5 yards of cloth cost 195.375, how much will 45.25 yards cost? Ans. $346.695+.

13. If an imperial gallon contains 277.274 cubic inches, how many cubic inches in 328.55 gallons?

Ans. 91098.3727.

14. A gallon of distilled water weighs 8.33888 pounds; how many gallons in 1000 pounds of such water?

Ans. 119.92+.

15. A cubic inch of water weighs 252.458 grains; how many cubic inches in 157786.25 grains?

Ans. 625cu. in.

16. A drew 41.25 barrels, of 31.5 gallons each, from a cistern containing 2000 gallons; how much remained?

Ans. 700.625.

17. A bought 78.25 acres of land at $128.50 an acre, and sold it for $9781.25; what was the loss on each acre? Ans. $3.50.

HISTORICAL, GEOGRAPHICAL, ETC. PROBLEMS.

The Battles of the Revolution.

1. At the battle of Lexington, the Americans lost 90 men, the British 190 more; how many did the British lose?

2. At the battle of Bunker Hill, the Americans had 1500 men, the British 1500 more; how many had the British?

3. In this battle the Americans lost 450 men, the British 604 more; how many did the British lose?

4. At the battle of Long Island, the British lost 367 men, the Americans 1233 more; how many did the Americans lose?

5. At the battle of Trenton, the British lost 45 in killed and wounded, and 1000 prisoners; what was their loss?

6. In the battle of Brandywine, the British lost 800 men, and the Americans 450 more; how many did the Americans lose?

7. In the battle of Germantown, the British lost 600 men, and the Americans lost 600 more; how many did the Americans lose?

8. At the battle of Bennington, the Americans lost about 100, and the British 600 more; required the British loss.

9. At the battle of Monmouth, the Americans lost 70 in killed, and the British 230 more; required the British loss.

10. In taking Stony Point, Gen. Wayne lost 15 killed and 83 wounded, and the British lost 500 more in killed wounded, and prisoners; required the British loss.

11. At the battle of King's Mountain, the Americans lost 20 men, and the British 280 more; how many did the British lose?

12. At the battle of Guilford, the Americans lost 400 men, the British lost 100 more; required the British loss.

13. At Hobkirk's Hill, the British loss was about 258 men, and the Americans 8 more; how many did the latter lose?

14. At Yorktown, Washington had 11,000 Americans and 5000 French, and the British had 2000 more than the French; what was the force on each side?

15. At Yorktown, the Americans lost about 75 killed and 225 wounded; the British lost 156 killed, 170 more than this wounded, and 70 missing; what was the loss on each side, not including prisoners?

PROBLEMS IN AMERICAN HISTORY.

1. America was discovered by Columbus in 1492, and Jamestown was settled in 1607; what was the difference of time?

2. Plymouth was settled in 1620; how long was that after America was discovered, and how long after the settlement of Jamestown?

3. The battle of Lexington was fought in 1775; how long was that after the settlement of Plymouth?

4. The Declaration of Independence was made in 1776; how long was that after the settlement of Jamestown?

5. The inauguration of Washington took place in 1789; how long was that after the battle of Bunker Hill, in 1775?

6. The battle of New Orleans took place in 1815; how long was that after the inauguration of Washington?

7. The frigate Constitution captured the British frigate Guerriere in 1812; how long was it after the Declaration of Independence?

8. Commodore Perry won his great naval victory in 1813; how long was that after the battle of Lexington?

9. General Jackson won his great victory at New Orleans in 1815; how long is it from then till the present?

10. General Packenham had 12000 men, and General Jackson 6000; how many more had the British than the Americans?

11. The British lost 1700 in killed and wounded, the Americans lost 13 men; what was the difference?

INTRODUCTION TO DENOMINATE NUMBERS.

(SUGGESTION TO TEACHERS.)

The first lessons in denominate numbers should be given in the concrete by the use of the *actual measures*. If these measures are not in the school, they can be procured at a trifling expense.

We should teach first a few of the simpler and more practical measures, rather than have the pupils commit the tables abstractly. Use these in little problems; as, "If there are 3 feet in 1 yard, how many feet are there in 2 yards?"

In respect to the order, teach first the *cent, dime,* and *dollar;* then the *foot, inch,* and *yard;* then the *pound* and *ounce;* then the *pint, quart,* and *gallon,* liquid measure; then the *pint, quart,* etc., dry measure; then the simpler units of *surface, volume,* and *time.*

Pupils should be required to make a *practical application* of these measures by finding the length of rooms, the height of ceilings, the weight of objects, etc.

Measures of Value.—In teaching the measures of value, show the pupils the *cent,* the *dime,* and the *dollar,* and illustrate their relative value. In teaching English money, it will be well to have the *penny, half-penny, shilling, sixpence,* etc., and similarly if we teach French and German money.

Measures of Length.—Have the foot-rule, and give definite ideas of the *foot* and *inch.* Have also the yard-stick, and give definite idea of the *yard.* Require pupils to draw the *inch, foot,* and *yard* on the board. Have the length of a *rod* marked on the floor or walls of the schoolroom. Show distance of a *mile, half-mile,* etc.

Measures of Weight.—Have the *ounce* and *pound* weight in the school-room, and let the children see and handle them and get a definite idea of their weight and relation. If there is a pair of scales, weigh objects, and have pupils "heft" books, chairs, etc., and learn to judge of their weight. The teacher can make these weights with little bags of sand.

Liquid Measure.—Have tin or wooden measures of the *gill, pint, quart,* and *gallon* in the school-room, and let pupils see and handle them until they become familiar with them. Let them see by actual trial how many *gills* make a *pint,* how many *pints* make a *quart,* etc. They can make their own table, and then commit it to memory.

Dry Measure.—Have also the measures of the *pint*, *quart*, *peck*, and *bushel* for the pupils to see and handle. With some sand or saw-dust they can see by actual trial how many pints make a quart, how many quarts make a peck, etc.

Surface Measure.—The teacher may make on the board a *square inch*, a *square foot*, and a *square yard*, and thus give definite ideas of them. Pupils can be led to see that a square yard contains 9 sq. ft., and a square foot contains 144 sq. in. Give also an idea of a *square rod*, or *perch*, and an *acre*. If the school is in the country, a *perch* and an *acre* may be measured off in a field adjoining or near the schoolhouse.

Volume Measure.—The teacher may show the pupils a *cubic inch*, and also a *cubic foot* if he has a model of it. He can draw these on the board, and teach the pupils to draw them. He can lead them to see that a cubic yard contains 27 cu. ft., and a cubic foot contains 1728 cu. in. By taking little sticks 4 inches long, the *cord* and *cord foot* may also be illustrated.

Time Measure.—Time may be taught by beginning with the *day*, from which we pass to the *week* and *year* and the number of days in each. By means of a clock or watch, the *hour*, the *minute*, and the *second* may be taught, the *number of hours* in a *day*, *number of minutes* in an *hour*, etc. The *month* is taught more arbitrarily, and the number of days in each month may be remembered by the stanza "Thirty days hath September," etc.

Show that the calendar begins one day later each common year, and two days later after each leap-year.

Circular Measure.—In teaching circular measure, draw a circle on the board, and divide it into *quadrants*; then show that a quadrant may be divided into 60 equal parts called *degrees*, and each degree into 60 equal parts called *minutes*, etc. Show also that the circumference contains 360°, etc. Show that these are *parts of the circumference*, and not *fixed lengths*, but that they differ in size in different circles.

NOTE.—A drill of this kind should precede the study of the text-book; but if it has not been previously presented, it is suggested that it be given in connection with the study of the subject in the book. Such a drill will give pupils definite ideas of what they are studying, and the "tables" will no longer be a list of abstract names and numbers.

SECTION VII.

DENOMINATE NUMBERS.

137. A ***Concrete Number*** is one which refers to some particular unit, as 2 books, 3 pounds, etc.

138. Concrete numbers are of two kinds; those in which the unit is *natural*, and those in which it is *artificial.*

139. A ***Denominate Number*** is a concrete number in which the unit is a *measure*, as 3 *pounds*, 4 *yards*, 5 *minutes*, etc.

140. ***Reduction*** is the process of changing a number from one denomination to another without changing its value.

141. ***Reduction Descending*** is the process of reducing from a higher to a lower denomination.

142. ***Reduction Ascending*** is the process of reducing from a lower to a higher denomination.

ENGLISH MONEY.

143. ***English***, or ***Sterling Money***, is the money of England.

TABLE.

4 farthings (far., or qr.)	equal	1 penny, . . .	d.
12 pence	"	1 shilling, . . .	s.
20 shillings	"	1 pound,* . . .	£.
21 shillings	"	1 guinea. . . .	g.

Mental Exercises.—Repeat the table of English Money.
How many far. in 2d.? in 3d.? in 6d.? in 8d.?
How many pence in 12far.? in 16far.? in 20far.? in 28far.?
How many pence in 2s.? in 3s.? in 5s.? in 6s.?
How many far. in 1s.? in 2s.? in 3s.? in 5s.?

* The £ coined in gold is called a sovereign. Its value is $4.8665. A five-shilling piece in silver is called a crown. A two-and-a-half-shilling piece in silver is called a half-crown.

REDUCTION DESCENDING.

1. How many farthings in 8 pence and 3 farthings?

SOLUTION.—In one penny there are 4 farthings, hence 4 times the number of pence equal the number of farthings; 4 times 8 are 32, and 32far. plus the 3far. equal 35 farthings.

OPERATION.

d.	far.
8	3
4	
32	
3	
35far. Ans.	

2. How many farthings in 14d. 2far.? Ans. 58far.
3. How many pence in 15s. 9d.? Ans. 189d.
4. How many shillings in £23 10s.? Ans. 470s.
5. How many farthings in 23s. 10d. 3far.?
6. How many pence in £32 19s. 3d.?

REDUCTION ASCENDING.

1. How many shillings, pence, and farthings, in 1487 farthings?

SOLUTION.—There are 4 farthings in one penny, hence in 1487far. there are as many pence as 4 is contained times in 1487, which are 371 pence, and 3far. remaining. There are 12 pence in one shilling, and in 371 pence there are as many shillings as 12 is contained times in 371, which are 30 shillings, and 11 pence remaining. Hence, 1485far. equal 30s., 11d., 3far.

OPERATION.

4)1487
12)371-3far.
30-11d.

2. How many shillings, pence, and farthings in 989 farthings? Ans. 20s. 7d. 1far.
3. Reduce 2676 farthings to shillings and pence. Ans. 55s. 9d.
4. How many pounds in 3178 farthings? Ans. £3 6s. 2d. 2far.
5. How many pounds in 9761 pence?
6. How many guineas in 17654 farthings?

TROY WEIGHT.

144. ***Troy Weight*** is used in weighing gold, silver, jewels, etc.

TABLE.

24 grains (gr.) .	equal	1 pennyweight,	pwt.
20 pennyweights .	"	1 ounce, . . .	oz.
12 ounces	"	1 pound, . . .	lb.

MENTAL EXERCISES.—How many oz. in 2lb.? in 3lb.? in 5lb.? How many lb. in 36oz.? in 48oz.? in 60oz.? in 1pwt.? in 2pwt.? How many pwt. in 2oz.? in 3oz.? in 4oz.? in 48grs.? in 72grs.?

1. How many grains in 15oz. 16pwt. 13grs.?

```
oz.  pwt.  grs.
15   16    13
20
----
300
 16
----
316 pwt.
 24
----
1264
632
----
7584
  13
----
7597grs. Ans
```

2. How many pennyweights in 7597grs.?

```
   grs.
24)7597
   -----
2|0)31|6 — 13grs.
     ----
     15 — 16pwt.
```

Ans. 15oz. 16pwt. 13grs.

NOTE.—In dividing by large numbers, like 24, we, of course, divide by Long Division. We indicated the result above.

How many

3 Grains in 6pwt. 12grs.? Ans 156grs

4 Pounds, oz., and pwts. in 963pwts.? Ans 48oz. 3pwt

5. Grains in 3oz. 11pwt. 14gr.?

6. Oz., pwt., and grs. in 5170grs?

7. Pounds, oz. etc., in 15786grs.?

APOTHECARIES WEIGHT..

145. ***Apothecaries Weight*** is used in mixing medicines. Medicines are bought and sold by Avoirdupois Weight.

TABLE.

20 grains (gr.) . .	equal	1 scruple, ...	℈.
3 scruples . . .	"	1 dram, ...	ʒ.
8 drams	"	1 ounce, ...	℥.
12 ounces	"	1 pound, ...	℔.

Mental Exercises.—1. How many grs. in 2 scruples? in 3 scruples? in 4 scruples.

2. How many scruples in 40gr.? in 60gr.? in 80gr.? in 160gr.?

3. How many scruples in 2 drams? in 4 drams? in 2 ounces? in 4 ounces?

4. How many drams in 2 ounces? in 1℔? in 12℥? in 36℥? in 120gr.?

1. How many grains in 15ʒ 2℈ 12gr.?

OPERATION.

ʒ	℈	gr.
15	2	12
3		
47		
20		
952gr. Ans.		

Note.—When convenient, add in the numbers as we multiply.

2. How many drams in 952 grains?

OPERATION.

2|0)95|2
3)47 — 12gr.
15 — 2℈
Ans. 15ʒ 2℈ 12gr

How many

1. Drams in 7℔. 5℥? Ans. 712ʒ.
2. Pounds and ounces in 239℥? Ans. 19℔. 11℥.
3. Scruples in 19℔. 8℥ 5ʒ 2℈? Ans. 56819℈.
4. Pounds, etc., in 92375gr.? Ans. 16℔. 3ʒ 1℈ 15gr

AVOIRDUPOIS WEIGHT.

146. ***Avoirdupois Weight*** is used for weighing every thing except jewels, precious metals, etc.

TABLE.

16 ounces (oz.) . .	equal	1 pound,	lb.
25 pounds . . .	"	1 quarter,	qr.
4 quarters . . .	"	1 hundred-weight,	cwt.
20 hundred-weight, .	"	1 ton,	T.

NOTE.—The *dram* was used as $\frac{1}{16}$ of an ounce, but is now obsolete.

MENTAL EXERCISES.—Ask mental questions upon this and the following tables, similar to those suggested under the previous tables.

1. How many ounces in 12lb. 13oz.?

OPERATION.

$$\begin{array}{rr} \text{lb.} & \text{oz.} \\ 12 & 13 \\ \underline{16} & \\ 72 & \\ 12 & \\ \underline{13} & \\ 205 & \text{oz.} \end{array}$$

2. How many pounds in 205 ounces?

OPERATION.

$$\begin{array}{l} \text{ounces.} \\ 16\underline{)205} \\ \quad\ 12 - 13\text{oz.} \end{array}$$

Ans. 12lb. 13oz.

How many

3 Ounces in 17lb. 12oz.?
4. Ounces in 3qr. 15lb. 13oz.?
5. Pounds in 5cwt. 3qr. 16lb.?
6. Pounds in 979 ounces?
7. Quarters in 27392 ounces?
8. Ounces in 20cwt. 3qr. 14lb. 11oz.?

NOTE.—Pupils may omit the next table until review, unless otherwise directed.

WINE MEASURE.

147. ***Wine Measure*** is used for measuring nearly all kinds of liquids.

TABLE.

4 gills (gi.)	equal	1 pint,	pt.
2 pints	"	1 quart,	qt.
4 quarts	"	1 gallon,	gal.

NOTE.—The *wine gallon* contains 231 cubic inches, while the *beer gallon*, used in measuring beer, and sometimes milk, contains 282 cu. in. In the old tables were given $31\frac{1}{2}$ gals. = 1 barrel; 63 gals. = 1 hogshead: 2 hogsheads = 1 pipe; 2 pipes = 1 tun. These are not measures, however, but vessels of variable capacity.

How many

1. Pints in 4gal. 3qt. 1pt.?
2. Gallons in 976 pints?
3. Gills in 17gal. 2qt. 3gi.?
4. Gallons in 1763 gills?

DRY MEASURE.

148. ***Dry Measure*** is used in measuring dry substances, as grain, fruit, salt, coal, etc.

TABLE.

2 pints (pt.) . . .	equal	1 quart,	qt.
8 quarts	"	1 peck,	pk.
4 pecks	"	1 bushel,	bu.

NOTE.—A *chaldron* of 36 bushels is sometimes used for measuring coal.

How many

1. Pints in 3pk. 6qt. 1pt. of berries?
2. Bushels in 314 quarts of clover seed?
3. Bushels in 3157 pints of cranberries?
4. What cost 4 bushels of berries at 2 cents a pint?

APOTHECARIES' FLUID MEASURE.

149. ***Apothecaries' Fluid Measure*** is used for measuring liquids in preparing medical prescriptions.

TABLE.

60 minims (♏)	.	equal	1 fluidrachm,	f℥.
8 fluidrachms	.	"	1 fluidounce,	f℥.
16 fluidounces	.	"	1 pint,	O.
8 pints . . .	.	"	1 gallon,	Cong.

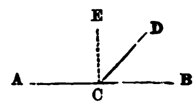
NOTE.—O is the initial of *octans*, the Latin for one-eighth, the pint being ⅛ of a gallon. Cong. is the abbreviation of *congiarium*, the Latin for gallon.

How many

1. Minims in 2O. 5f℥?
2. Pints in 8000 minims?
3. Fluidounces in 3Cong. 7O. 5f℥?
4. Gallons in 78561 minims?

MEASURE OF LENGTH.

150. ***Measure of Length***, or ***Long Measure***, is used for measuring length, breadth, height, distances, etc.



1. A ***Line*** is that which has length without breadth or thickness.
2. An ***Angle*** is the opening between two lines which diverge from a common point. Thus, ACD and DCB are angles.
3. A ***Right Angle*** is formed by one line perpendicular to another, as ACE or ECB.

TABLE.

12 inches (in.)	.	equal	1 foot,	ft.
3 feet	.	"	1 yard,	yd.
$5\frac{1}{2}$ yards, or $16\frac{1}{2}$ ft. .	.	"	1 rod,	rd.
320 rods	.	"	1 mile,	mi.
3 miles	.	"	1 league,	l.

NOTES.—1. In the old tables we also had 40 rods = 1 furlong and 8 furlongs = 1 mile.

2. *Cloth Measure* is not now used. Cloth, muslin, etc. are bought and *sold by the* yard, half-yard, quarter, eighth, etc.

1. How many feet in 12rd. 3yd. 2ft.?

OPERATION.

```
rd. yd. ft.
12   3   2
 5½
---
63
 6
---
69yd.
 3
---
209ft. Ans.
```

NOTE.—We multiply by 5, and add to the product the 3 yds., and then multiplying by ½, we have 69 yd.

2. How many rods in 209 ft.?

OPERATION.

```
  feet.
 3)209
   ---
5½)69ft.
 2   2
--  ---
11)138
   ---
    12 — 6 halves = 3yd.
```

Ans. 12rd. 3yd. 2ft.

NOTE.—To divide by 5½, we reduce both to halves, then the remainder is *halves*, which we reduce to *wholes*, by dividing by 2.

How many

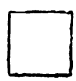
3. Feet in 16rd. 5yd. 2ft.? Ans. 281ft.
4. Inches in 17yd. 1ft. 11in.? Ans. 635in.
5. Yards in 3146 inches? Ans. 87yds. etc.
6. Rods in 6547 inches?
7. Feet in 312rd. 4yd.?
8. Rods in 4389 feet?
9. Miles in 19280 feet?
10. Inches in 2m. 244rd. 8in.?
11. How many inches from New York to Philadelphia, if the distance is 96 miles?

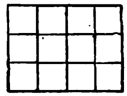
SURFACE OR SQUARE MEASURE.

151. ***Surface*** or ***Square Measure*** is used in measuring surfaces, as land, boards, etc.

1. A ***Surface*** is that which has length and breadth without thickness.

2. A ***Square*** is a surface which has four equal sides and four right angles, as in the margin.





3. A ***Rectangle*** is a surface which has four sides and four right angles. A slate, a door, the sides of a room, etc., are examples of rectangles.

4. The ***Area*** of a surface is expressed by the number of times it contains a small square as a *unit of measure.*

5. *The area of a square or rectangle is equal to the length multiplied by the breadth.* For, in the rectangle above, the whole number of little squares is equal to the number in each row multiplied by the number of rows: that is, 4×3 which equals 12, which is the same as the number of units in length multiplied by the number in breadth.

TABLE.

144 square inches (sq. in.)	equal	1 square foot,	sq. ft.
9 square feet	"	1 square yard,	sq. yd.
$30\frac{1}{4}$ square yards . . .	"	1 perch, or sq. rod,	P.
160 perches	"	1 acre,	A.
640 acres	"	1 square mile,	sq. mi.

NOTE.—The *rood,* equal to 40 perches, is found only in old title deeds and surveys.

1. How many square feet in 28P. 18sq. yd. 5 sq. ft.?

OPERATION.

P. sq. yd. sq. ft.
28 18 5
$30\frac{1}{4}$
858
7
865sq. yd.
9
7790sq. ft.

NOTE.—We multiplied by 30, added in the 18 sq. yds., and then multiplied by $\frac{1}{4}$ and took the sum.

2. How many perches are there in 7790sq. ft.?

OPERATION

sq. ft.
9)7790
$30\frac{1}{4}$)865 — 5sq. ft.
4 4
121)3460
28 — 72 fourths,
or 18sq. yd.
Ans. 27P. 18sq. yd. 5sq. ft.

NOTE.—To divide by $30\frac{1}{4}$ we reduce both divisor and dividend to 4ths, and then divide; the remainder is 72 fourths, or 18 sq. yd.

How many

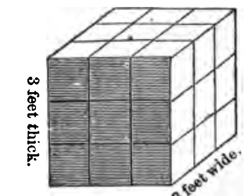
3. Square inches in 2sq. yd. 3sq. ft.?
4. Square feet in 2R. 13P. 16sq. yd.?
5. Perches in 8765 square feet?
6. Acres in 1997 perches? Ans. 12A. 1R. 37P.
7. Perches in 29A. 3R. 19P.? Ans. 4779P.
8. Acres in 89763 square yards?

Ans. 18A. 2R. 7P., etc.

CUBIC OR SOLID MEASURE.

152. ***Cubic*** or ***Solid Measure*** is used in measuring things which have length, breadth, and thickness.

1. A ***Volume*** is that which has length, breadth, and thickness. A volume is also called a *solid*.



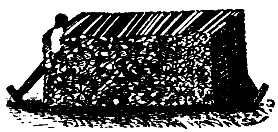
2. A ***Cube*** is a volume bounded by six equal squares. A ***Rectangular Volume*** is one bounded by rectangles. Cellars, boxes, rooms, etc., are examples of rectangular volumes.

3. The ***Contents*** of a volume are expressed by the number of times it contains a *cube* as a *unit of measure*.

The contents of a Cube or Rectangular Solid are equal to the product of the length, breadth, and height.

For in the volume above, the number of cubic units on the base is equal to the length multiplied by the breadth, and the whole number of cubic units equals the number on the base multiplied by the number of layers; hence the whole number equals $3 \times 3 \times 3 = 27$.

4. A *cord of wood* is a pile 8 feet long, 4 feet wide, and 4 feet high. A *cord foot* is a part of this pile, 1 foot long; it equals 16 cubic feet.



1 cord.

TABLE.

1728 cubic inches (cu. in.)	equal	1 cubic foot,	cu. ft.
27 cubic feet	"	1 cubic yard.	cu. yd
16 cubic feet	"	1 cord foot,	cd. ft.
8 cord feet, or 128 cubic feet, . . .	"	1 cord of wood,	Cd.

NOTE.—The *ton* of 40 ft. for *round* timber and 50 ft. for *hewn* timber is seldom used.

How many

1. Cu. in. in 7cu. ft. 96cu. in.?
2. Cu. in. in 12cu. yd. 25cu. ft.?
3. Cu. ft. in 8469 cubic inches?
4. Cu. yd. in 60463 cubic inches?
5. Cords in 8192 cubic feet?

MEASURE OF TIME.

153. ***Time*** is the measure of duration.

TABLE.

60 seconds (sec.) .	equal	1 minute,	min
60 minutes . . .	"	1 hour,	h.
24 hours	"	1 day,	da.
7 days	"	1 week,	wk.
4 weeks	"	1 month,	mo
52 weeks	"	1 year,	yr.
365 days	"	1 common year,	yr.
100 years	"	1 century,	cen.

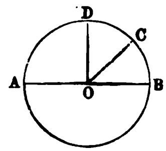
How many

1. Minutes in 1 day?
2. Seconds in 1 day?
3. Hours in 1 year?
4. Minutes in 1 year?
5. Hours in 56780 seconds?
6 Days in 600000 seconds?

CIRCULAR MEASURE.

154. ***Circular Measure*** is used to measure angles and directions, latitude and longitude, etc.

1. A ***Circle*** is a figure bounded by a curve line, every point of which is equally distant from a point within, called the *centre*.



2. The ***Circumference*** is the bounding line; any part of the circumference, as BC, is an *arc;* AB is the *diameter*, and OC the *radius*.

3 For the purpose of measuring angles, the circumference is divided into 360 equal parts, called *degrees;* each degree into 60 equal parts, called *minutes;* each minute into 60 equal parts, called *seconds.*

4. Any angle at the centre, as COB, is measured by the arc BC included between its sides. A right angle is measured by 90 degrees; half a right angle, by 45 degrees, etc.

TABLE.

60 seconds (″) . .	equal	1 minute, . .	′
60 minutes . . .	"	1 degree, . . .	°
30 degrees . . .	"	1 sign	S.
12 signs, or 360°, .	"	1 circumference,	C.

How many

1. Seconds in 24′ 32″? Ans. 1472″.
2. Seconds in 23° 24′ 15″? Ans. 84255″.
3. Minutes in 1472″? Ans. 24′ 32″.
4. Degrees in 84255″? Ans. 23° 24′ 15″.

5. What is the difference between the number of minutes in a day and the number of minutes in a circumference?

6. What is the difference between the number of seconds in a day and the number of seconds in a circumference?

7. If you study 6 hours a day, for 5 days in a week, how many minutes will you study in a week?

MISCELLANEOUS TABLES.

155. COUNTING.

12 units	equal	1 dozen.
12 dozen	"	1 gross.
12 gross	"	1 great gross.
20 units	"	1 score.

156. PAPER.

24 sheets	equal	1 quire.
20 quires	"	1 ream.
2 reams	"	1 bundle.
5 bundles	"	1 bale.

157. WEIGHT, CAPACITY, LENGTH, ETC.

56 pounds of rye or corn	equal	1 bushel.
60 pounds of wheat or clover seed	"	1 bushel.
60 pounds of beans or potatoes	"	1 bushel.
100 pounds of fish	"	1 quintal.
196 pounds of flour	"	1 barrel.
220 pounds of shad or salmon	"	1 barrel.
200 pounds of other fish	"	1 barrel.
200 pounds of beef or pork	"	1 barrel.
14 pounds (by English law)	"	1 stone.
8 bushels of wheat	"	1 quarter.
4 inches	"	1 hand.
9 inches	"	1 span.
22 inches (in Scripture)	"	1 cubit.

A knot or nautical mile is 6086.7 feet.

A surveyor's chain of 100 links is 4 rods long.

1. How many units in a gross?
2. How many pins in a great gross?
3. How many sheets in a ream? In a bundle?
4. How many sheets in a bale? In 12 bales?
5. How many bushels of rye will weigh as much as 14 bushels of wheat? Ans. 15bu.
6. How many bushels of beans will weigh as much as 30 bushels of corn? Ans. 28bu.
7. If Dr. Windship lifts 3000 pounds, how many barrels of beef can he lift? Ans. 15 barrels.

MISCELLANEOUS PROBLEMS.

1. Reduce 967 pence to pounds.
2. Reduce 1840 pence to pounds.
3. Reduce 2480 farthings to shillings.
4. How many pounds in 8000 grains Troy?
5. How many pounds in 10000 ounces Avoirdupois?
6. Reduce £12 9s. 6d. to pence.
7. Reduce 8lb. 7oz. 13pwt. to penyweights.
8. Reduce 12cwt. 14lb. 15oz. to ounces.
9. How many seconds in 24 hours, or one day?
10. How many pounds in 16cwt. 3qr. 13lb.?
11. How many tons in 9876 pounds?
12. How many tons in 165762 ounces?
13. Change 6ʒ 2℈ 12gr. to grains.
14. Reduce 9℥ 4ʒ 1℈ 10gr. to grains.
15. How many pounds in 58763ʒ?
16. How many pounds in 76542℈ ?
17. Reduce 3mi. 284rd. 2yd. to yards.
18. Reduce 47692 feet to miles.
19. Reduce 1234560 inches to miles.
20. Adam died at the age of 930 years; how many seconds was this?
21. Methuselah died at the age of 969 years; how many seconds was this ?
22. If the pulse beat 75 times a minute, how often does it beat in a day? Ans. 108000 times.
23. How long will it take to count a million, at the rate of a hundred a minute, working 12hrs. a day? Ans. 13d. 10h. 40m.
24. If the distance around the earth is 25000 miles, how long will it take to walk the distance, walking 4 miles an hour? Ans. 260d. 10h.
25. If £1 equals $4.8665, what is the value of £5 in United States Money?

26. How many times will a clock that ticks seconds, tick in one day? Ans. 86400 times.

27. A little girl picked $2\frac{1}{2}$ pecks of berries and sold them at 5 cents a pint; what did she receive? A. $2.00.

28. How many crayons are there in 25 boxes, if each box contains one gross? Ans. 3600.

29. How many vials, holding 2 gills each, can be filled from a gallon of brandy? Ans. 16.

30. If you are 10 years old, how many minutes have you lived, allowing $365\frac{1}{4}$ days to a year? A. 5259600 min

31. How many doses of medicine, of 6 gr. each, can be made from 4 drams? Ans. 40.

32. If £1 equals $4.8665, required the value of £7 15s. in the money of the United States. Ans. $37.715.

33. If £2 equals $9.733, what is the value of $37.96 in English Money? Ans. £7 16s.+.

34. If 12 of Henry's peaches fill a quart measure, how many will there be in a bushel? Ans. 384.

35. How many square rods in a rectangular field 32 rods long and 12 rods wide? Ans. 384 sq. rods.

36. How many square feet in a board 18 feet long, and $2\frac{1}{2}$ feet wide? Ans. 45 sq. feet.

37. How many cubic feet in a block of stone 6 feet long, 3 feet wide, and 2 feet thick? Ans. 36 cu. feet.

38. Required the value of a rectangular lot 36 rods long, and 20 rods wide, at $3 a square rod? Ans. $2160.

39. How many cords in a pile of wood 48 feet long, 4 feet wide, and 4 feet high? Ans. 6 cords.

40. How many cords in a pile of wood 16 feet long, 8 feet high, and 4 feet wide? Ans. 4 cords.

41. What must I pay for a pile of wood 24 feet long, 12 feet high, and 4 feet wide, at $1.50 a cord?

Ans. $13.50.

42. How much time is wasted by taking an hour's nap each afternoon, for 24 years of 365 days each?

Ans. 1 year.

43. When apples sell at 16 cents a half-peck; what are they worth a bushel? Ans. $1.28.

44. What will it cost to pave 75 sq. yards of walk at 50 cents a sq. foot? Ans. $337.50.

45. At 5 cents a half-pint, how much does a milkman receive for 25 gallons of cream? Ans. $20.00.

46. A grocer sold 8 bushels of chestnuts at 6 cents a quart; what did he receive for them? Ans. $15.36.

47. What will 12 pounds of drugs cost at the rate of 32 cents a dram? Ans. $983.04.

48. A grocer bought 132 eggs at 18 cents a dozen; what did they cost? Ans. $1.98.

49. How many steps of 3 feet each will a person take in walking 2½ miles? Ans. 4400.

50. A grocer bought 16 barrels of beef at 10½ cents a pound; what did it cost him? Ans. $336.

51. How much will 2 A. 20 P. of land cost at $2½ a perch? Ans. $850.

52. A man bought 32 reams of paper at 18¾ cents a quire; what was the cost? Ans. $120.

53. How many acres are there in a lot of land 160 rods long and 80 rods wide? Ans. 80 acres.

54. What cost 2 barrels of alcohol, each containing 31½ gallons, at 3½ cents a gill? Ans. $70.56.

55. How much will I get for 16 gross of pins at 1½ cents for each pin? Ans. $34.56.

56. If Dr. Davis use 2℥. 5ʒ. 2℈. of drugs daily, how much will he use in a week? Ans. 1℔. 6℥. 7ʒ. 2℈.

57. If blackberries are worth $3.20 a bushel, what are they worth a quart? Ans. 10 cents.

58. How many half-pint bottles will two gallons of ink fill? Ans. 32

59. What will 20 gross of lead-pencils cost at 62½ cents a dozen? Ans. $150.

60. How many quart baskets will 2 bu. 2 qts. of strawberries fill? Ans. 66.

61. How many ounces of calomel will it take to make 384 pills of 5 grains each? Ans. 4 oz.

62. Dr. Hess made calomel pills of 5 grains each, in all 2℥. 2ʒ. 1℈.; how many pills did he make? Ans. 220.

63. At $1.50 a bushel, what will a farmer receive for 2400 pounds of clover-seed? Ans. $60.

64. How many panels, of 10 feet, of fence will enclose a field which is 40 rods long and 30 rods wide? Ans. 231.

65. What will 2 bu. 3 pk. 6 qt. of shellbarks cost at 12 cents a quart? Ans. $11.28.

66. How many sheets of paper in 2½ reams and 3½ quires? Ans. 1284.

67. A farmer put up 1⅔ miles of fence at $1½ a rod; what did it cost? Ans. $800.

68. How many pounds in a load of corn which, at 75 cents a bushel, cost $32.25? Ans. 2408 lbs.

69. How many bushels of grapes at 15 cents a quart can be bought for $8.80? Ans. 1 bu. 3 pk. 2 qt. 1⅓ pt.

70. How many miles of fence, at $1.50 a rod, can be put up for $1200. Ans. 2 miles, 160 rods.

71. A man sold 2520 lbs of wheat at $1.87½ per bushel; what did he get for the whole? Ans. $78.75.

72. What will 36 packages of paper cost at 16⅔ cents a quire, if each package contains 2 reams? Ans. $240.

ADDITION OF DENOMINATE NUMBERS.

158. ***Addition of Denominate Numbers*** is the process of finding the sum of two or more denominate numbers of the same kind of quantity.

1. Find the sum of £8 7s. 5d.; £9 8s. 6d.; £7 14s. 9d.

SOLUTION.—We write the numbers so that units of the same kind shall stand in the same column, and begin at the right to add. 9d plus 6d. plus 5d. equal 20d., which, by reduction, we find equal 1s. and 8d. We write the 8d. under the pence column, and reserve the 1s. to add to the column of shillings. 1s. plus 14s. plus 8s.

OPERATION.

£	s.	d.
8	7	5
9	8	6
7	14	9
25	10	8

plus 1s. equal 30s., which, by reduction, we find equal £1 and 10s. We write the 10s. in shillings column, and add the £1 to the column of pounds, etc. Hence the following

RULE.—1. *Write the numbers so that units of the same name stand in the same column, and commence at the right to add.*

2. *Add as in simple numbers, reduce by division the sum of each column to the next higher denomination, write the remainder under the column added, and add the quotient to the next column.*

3. *Proceed in the same manner with all the columns to the last, under which write the entire sum.*

PROOF.—The same as in simple numbers.

(2.)

£	s.	d.
24	12	6
25	13	9
17	18	10

(3.)

£	s.	d.
25	16	8
17	13	9
14	17	11

(4.)

£	s.	d.
123	14	6
137	18	10
246	19	11

(5.)

lb.	oz.	pwt.
17	9	16
25	6	12
72	11	13
57	10	19

(6.)

lb.	oz.	pwt.
18	9	16
36	8	21
29	7	23
42	11	17

(7.)

lb.	oz.	pwt.
92	7	12
71	3	17
28	9	10
36	11	18

(8.)

cwt.	qr.	lb.	oz
20	3	12	11
16	2	16	12
17	0	22	20
19	1	18	19

(9.)

qr	lb.	oz.	dr.
12	16	12	11
13	23	9	10
14	24	14	15
15	16	15	8

(10.)

rd.	yd.	ft.	in
17	4	2	6
21	2	1	7
23	3	0	8
25	5	2	9

(11.)					(12.)			(13.)			
lb.	℥	ʒ	℈	gr.	gal.	qt.	pt.	L.	mi.	rd.	yd.
28	11	7	2	16	36	2	1	16	2	30	4
19	9	5	1	23	42	1	1	14	2	32	3
27	8	3	2	17	25	3	0	28	1	28	1
24	7	2	1	18	28	3	1	34	0	37	2

SUBTRACTION OF DENOMINATE NUMBERS.

159. ***Subtraction of Denominate Numbers*** is the process of finding the difference between two compound numbers of the same kind of measure.

1. From 10oz. 12pwt. 20gr. take 7oz. 15pwt. 16gr.

OPERATION.

oz.	pwt.	gr.
10	12	20
7	15	16
2	17	4

SOLUTION.—We write the subtrahend under the minuend, writing units of the same name in the same column, and commence at the lowest denomination to subtract. 16gr. subtracted from 20gr. leave 4grs., which we write under the grains. 15pwt. from 12pwt. we cannot take; we will therefore take 1oz. from the 10oz., leaving 9oz.; 1oz. equal 20pwt., which added to 12pwt. equal 32pwt.; 15pwt. subtracted from 32pwt. equal 17pwt., which we write under pwts. 7oz. from 9oz. (or since it will give the same result, we may add 1oz to 7oz. and say, 8oz. from 10oz.) leave 2oz. Hence the following

RULE.—1. *Write the subtrahend under the minuend, with units of the same denomination in the same column.*

2. *Commence at the lowest denomination, and subtract each number in the subtrahend from the corresponding number in the minuend.*

3. *If the number in the subtrahend exceeds the number in the minuend, add to the latter as many units of that denomination as make one of the next higher, and then subtract; add also one to the next number in the subtrahend before subtracting.*

4. *Proceed in the same manner with each denomination to the last.*

PROOF.—The same as in simple numbers.

(2.)

£	s.	d.	far
143	11	10	2
115	14	6	3
27	17	3	3

(3.)

£	s.	d.	far.
930	17	7	3
246	19	8	1

(4.)

lb.	oz.	pwt.	gr
16	10	16	18
13	11	17	15
2	10	19	3

(5.)

lb	oz.	pwt.	gr.
125	8	14	20
96	9	10	23

(6.)

cwt.	qr.	lb.	oz.
112	3	17	12
37	1	10	13

(7.)

T.	cwt.	qr.	lb.	oz
236	13	2	18	12
127	11	4	22	10

(8.)

hhd.	gal.	qt.	pt.
128	27	0	1
106	30	2	1

(9.)

yr.	mo.	wk.	da.	h.
216	10	2	5	16
123	10	3	2	20

(10.)

sq.yd	sq.ft.	sq.in
226	0	120
134	5	130

(11.)

A.	R.	P.
426	1	30
207	3	35

(12.)

L.	mi.	rd.	yd.
16	2	30	2
14	2	32	3

(13.)

S.	°	′	″
25	20	30	40
20	30	40	50

14. A farmer had 200 bu. of wheat, and sold 28bu. 2pk. 5qt. 1pt. to one man, and as much more to another; how much remained? Ans. 142bu. 2pk. 5qt.

15 A miner having 112lb. of gold sent his mother 17lb. 10oz. 15pwt. 20gr. and 3lb. 16pwt. less to his father; how much did he retain?

Ans. 79lb. 3oz. 4pwt. 8gr.

16. Subtract 16dol. 57cts. 5½ mills from $25 20cts. 7½ mills, and add 2 eagles and 25¾ dimes to the result.

Ans. 31dol. 20cts. 7 mills.

17. Add 96lb. 9oz. 10pwt. 23gr. to 125lb. 8oz. 14pwt. 20gr. and subtract the sum from the sum of 102lb. 11oz. 16pwt. and 256lb. 9oz. 19pwt.

MULTIPLICATION OF DENOMINATE NUMBERS

160. ***Multiplication of Denominate Numbers*** is the process of multiplying a denominate number by an abstract number.

1. Multiply £12 11s. 7d. by 8.

OPERATION.		
£	s.	d.
12	11	7
		8
100	12	8

SOLUTION.—8 times 7d. are 56d., which by reduction we find is equal to 4s. and 8d. We write the 8 pence under the pence, and reserve the 4s. to add to the next product. 8 times 11s. are 88s., which added to the 4s. equal 92s., which we find by reduction equal £4 and 12s. 8 times £12 are £96, which added to £4 equal £100. Hence the following

RULE.—*Write the multiplier under the lowest denomination of the multiplicand, multiply as in simple numbers, reducing as in addition.*

PROOF.—The same as in simple numbers.

EXAMPLES FOR PRACTICE.

(2.)

cwt.	qr.	lb.	oz.
18	3	21	9
			5

(3.)

lb.	oz.	pwt.	gr.
16	8	15	17
			3

(4.)

M.	da.	h.	min.	sec.
50	10	20	30	40
				7

(5.)

£	s.	d.	far.
13	12	9	2
			8

(6.)

hhd.	gal.	qt.	pt.
21	35	3	1
			9

(7.)

℔	℥	ʒ	℈	gr
12	8	7	2	20
				11

8. Multiply 12L. 2mi. 232rd. by 5, by 6, by 7, by 8.

9. Multiply 23ch. 18bu. 2pk. 7qt. 1pt. by 4, by 5, by 9, by 10.

10. A farmer sold 5 loads of hay, each containing 15cwt. 3qr. 15lb.; how much did he sell?

Ans. 79cwt. 2qr.

11. Multiply 13yr. 10mo. 3wk. 5da. by 5, and that product by 3. Ans. 208yr. 7mo. 3wk. 5da.

12. If a man walk 17mi. 300rd. in each of 21 days, how far will he walk in all? Ans. 376mi. 220rd.

13. If a farmer raise 60bu. 3pk. 6qt. 1pt. of grain on one acre, how much can he raise at the same rate on 48 acres? Ans. 2925bu. 3pk.

14. A owned 1000A. of land; he sold B 96A. 3R. 80P., and 4 times as much to C; how much remained?
Ans. 515A. 1R. 10P.

DIVISION OF DENOMINATE NUMBERS.

161. ***Division of Denominate Numbers*** is the process of dividing when one or both terms is a denominate number. There are two cases.

Case I.

162. To divide a denominate number into equal parts.

1. Divide £103 7s. 6d. into 5 equal parts; that is, take $\frac{1}{5}$ of it.

Solution.—$\frac{1}{5}$ of £103 is £20 and £3 remaining. £3 equal 60s., which added to 7s. equal 67s.; $\frac{1}{5}$ of 67s. is 13s. and 2s. remaining. 2s. equal 24d., which added to 6d. equal 30d.; $\frac{1}{5}$ of 30d. is 6d. Hence the following

OPERATION.

	£	s.	d.
5)	103	7	6
	20	13	6

Rule.—1. *Begin at the highest denomination, and divide as in simple numbers.*

2. *If there is a remainder, reduce it to the next lower denomination, add to it the number of that denomination, and divide as before, and thus continue to the last.*

EXAMPLES FOR PRACTICE.

(2.)

	£	s.	d
4)	61	18	4
	15	9	7

(3.)

	lb.	oz.	pwt.	gr.
6)	76	10	14	12

(4.)

	T.	cwt.	qr.	℔
7)	112	16	2	16
	16	2	1	13

(5.)

	cwt.	qr.	lb.	oz.	dr.
8)	125	3	19	12	8

(6.)

	hhd.	gal.	qt.	pt	gi
9)	108	42	2	1	2

(7.)

	mi.	rd.	yd.	ft.
11)	120	313	3	2

(8.)

	A.	P.	sq. yd.
5)	112	144	24

(9.)

	bu.	pk.	qt.	pt.
9)	1137	3	4	1

(10.)

	lb.	oz.	pwt.	gr.
8)	37	10	17	16

11. A miner divides 37lb. 10oz. 17pwt. 16gr. of gold among 9 sisters; how much does each receive?

Ans. 4lb. 2oz. 10pwt. 20$\frac{4}{9}$gr.

12. A man walked 376 mi. 276rd. in 22 days; what was the average distance each day?

Ans. 17mi. 41$\frac{7}{11}$rd.

13. If 26 casks contain 21hhd. 11gal. 2qt. 1pt., what is the capacity of each cask? Ans. 51gal. 1qt. $\frac{17}{26}$pt.

Case II.

163. To divide a denominate number by a similar denominate number.

1. Divide £26 6s. 2d. by £4 15s. 8d.

SOLUTION.—£26 6s. 2d. we find by reduction equal 6314 pence; £4 15s. 8d. equals 1148 pence; and dividing 6314d. by 1148d. we obtain a quotient of 5½. From this solution we have the following

OPERATION.

£26 6s. 2d. = 6314d.

£4 15s. 8d. = 1148d.

1148)6314(5½, Ans.
6314

RULE.—*Reduce both dividend and divisor to the lowest*

denomination mentioned in either, and then divide as in simple numbers.

REMARK.—The division may also be made before the reduction to lower denomination; and this will be shorter when there is no remainder.

2. Divide £48 7s. 4d. by £6 11d. Ans. 8.
3. Divide 69bu. 3pk. 6qt. by 6bu. 3pk. 6qt.
Ans. $10\frac{3}{37}$.
4 Divide 80bu. 2pk. 4qt. by 13bu. 3pk. 5qt.
Ans. $5\frac{71}{89}$.
5. Divide 697lb. 7oz. 5dr. by 60lb. 10oz. 6dr.
Ans. $11\frac{1}{2}$.
6. A man travelled 3mi. 276rd. 4yd. in one hour; in what time will he travel 247mi. 110rd. 3yd.?
Ans. 64 hrs.
7. A drove of cattle ate 6T. 15cwt. 3qr. 12lb. of hay in a week; how long will 33T. 19cwt. 1qr. 10lb. last them? Ans. 5 weeks.

PROBLEMS IN TIME.

1. Washington was born Feb. 22d, 1732, and died Dec. 14th, 1799; what was his age?

SOLUTION.—We write the number of the year, month, and day of both periods, and subtract the one from the other, as is shown in the margin.

OPERATION.

yr.	mo.	da.
1799	12	14
1732	2	22
67	9	22

2. John Adams was born the 19th of October, 1735, and died the 4th of July, 1826; required his age.
3. Thomas Jefferson was born April 2d, 1743, and died July 4th, 1826; what was his age?
4. James Madison was born March 16th, 1751. and died June 28th, 1836; required his age.
5. James Monroe was born April 28th, 1758, and died July 4th, 1831; required his age.

6. John Quincy Adams was born July 11th, 1767, and died Feb. 23d, 1848; what was his age?

7. Andrew Jackson was born March 15th, 1767, and died June 8th, 1845; required his age.

8. Martin Van Buren was born Dec. 5th, 1782, and died July 24th, 1862; required his age.

9. William Henry Harrison was born Feb. 9th, 1773 and died April 4th, 1841, required his age.

10. James K. Polk was born Nov. 2d, 1795, and died June 15th, 1849; required his age.

11. General Zachary Taylor was born Nov. 24th, 1784, and died July 9th, 1850; required his age.

12. How long has a note to run which is dated Dec. 30th, 1862, and made payable Jan. 16th, 1864?

Ans. 1yr. 16da.

13. The Revolution was commenced the 19th of April, 1775, and terminated January 20th, 1783; how long did it continue? Ans. 7yr. 9mo. 1da.

PROBLEMS IN LATITUDE AND LONGITUDE.

1. The latitude of Boston is 42° 21′ 23″ north; that of Charleston, 32° 46′ 33″ north; what is the difference of latitude?

2. The latitude of New York is 40° 24′ 40″ N.; that of New Orleans, 29° 57′ 30″ N.; what is the difference of latitude?

3. The latitude of Philadelphia is 39° 56′ 39″; that of Savannah is 32° 4′ 56″; what is the difference of latitude?

4. The latitude of Baltimore is 39° 17′ 23″; that of St. Louis is 38° 37′ 28″; what is the difference of latitude?

5. The latitude of the Cape of Good Hope is 30° 55′ 15″ S.; that of Cape Horn, 55° 58′ 30″; what is the difference of latitude?

SECTION VIII.

PERCENTAGE.

164. ***Percentage*** is a process of computation in which the basis of comparison is a hundred.

165. The term *per cent.* means *by* or *on a hundred;* thus, 5 per cent. of any thing means 5 of a hundred of it.

166. Hence, 1 per cent. of a number is $\frac{1}{100}$ of it; 2 per cent. is $\frac{2}{100}$ of it; 5 per cent. is $\frac{5}{100}$ of it, etc. It is also evident that 100 per cent. of a number is the whole of it.

167. ***The sign of Percentage*** is %, and is read *per cent.* The per cent. is also indicated by a common fraction or a decimal; thus, $5\% = \frac{5}{100} = .05$.

168. In percentage there are four quantities considered:

1. The *Base*, or number on which percentage is estimated.
2. The *Rate*, or number denoting how many of a hundred.
3. The *Percentage*, denoting how many of the basis.
4. The *Amount* or *Difference* of the basis and percentage.

RATE EXPRESSED BY A FRACTION.

2 per cent.	equals	.02,	or	$\frac{2}{100}$,	or	$\frac{1}{50}$.
4 per cent.	"	.04,	"	$\frac{4}{100}$,	"	$\frac{1}{25}$.
5 per cent.	"	.05,	"	$\frac{5}{100}$,	"	$\frac{1}{20}$.
20 per cent.	"	.20,	"	$\frac{20}{100}$,	"	$\frac{1}{5}$.
25 per cent.	"	.25,	"	$\frac{25}{100}$,	"	$\frac{1}{4}$.
125 per cent.	"	1.25,	"	$\frac{125}{100}$,	"	$\frac{5}{4}$.

$\frac{1}{2}$ per cent. is $.00\frac{1}{2}$, or .005; $\frac{1}{5}$ per cent. is $.00\frac{1}{5}$, or .002; $12\frac{1}{2}$ per cent. is $.12\frac{1}{2}$, or .125, etc.

EXERCISES.

Express decimally		Express in a common fraction	
1. 6%.	Ans. .06.	6. 6%.	Ans. $\frac{3}{50}$.
2. 12%.	Ans. .12.	7. 12%.	Ans. $\frac{3}{25}$.
3. $16\frac{2}{3}$%.	Ans. $.16\frac{2}{3}$.	8. 50%.	Ans. $\frac{1}{2}$.
4. 24%.	Ans. .24.	9. $12\frac{1}{2}$%.	Ans. $\frac{1}{8}$.
5. $33\frac{1}{3}$%.	Ans. $.33\frac{1}{3}$.	10. $16\frac{2}{3}$%.	Ans. $\frac{1}{6}$.

Express as a per cent.

1. $\frac{1}{4}$.	Ans. 25%.	4. $\frac{1}{8}$.	Ans. $12\frac{1}{2}$%
2. $\frac{1}{5}$.	Ans. 20%.	5. $\frac{1}{6}$.	Ans. $16\frac{2}{3}$%.
3. $\frac{1}{2}$.	Ans. 50%.	6. $\frac{2}{3}$.	Ans. $66\frac{2}{3}$%.

Case I.

169. Given the base and rate, to find the percentage.

1. What is 5 per cent. of $250?

OPERATION.

$$\begin{array}{r} \$250 \\ .05 \\ \hline \$12.50 \end{array}$$

Solution.—5 per cent. of $250 is $\frac{5}{100}$ of $250, or .05 times $250, which, by multiplying, we find to be $12.50. Hence the following

Rule.—*Multiply the base by the rate, expressed decimally.*

2. What is 5 per cent. of $280? Ans. $14.
3. What is 6 per cent. of $190? Ans. $11.40.
4. What is 7 per cent. of $240? Ans. $16.80
5. What is 8 per cent. of 125yd.? Ans 10yd.
6. What is 9 per cent. of 364lb.?
7. What is 10 per cent. of 982ft.?
8. What is 12 per cent. of 831in.?
9. What is $12\frac{1}{2}$ per cent. of 320oz.?
10. What is $16\frac{2}{3}$ per cent. of 630yds.?
11. What is 35 per cent. of 1286 miles?
12. What is 40 per cent. of 2467 pounds?
13. What is 75 per cent. of 3182 perches?

14 A man bought a cow for \$30, and sold her at a gain of 25%; what did he gain? Ans. \$7½.

15. A man bought a horse for \$150, and sold him at a gain of 30%; how much did he gain? Ans. \$45.

16. A lady bought 360 acres of land, and sold 12½% of it; how much did she retain? Ans. 315 acres.

17. A man bought a horse for \$4250, and sold it at a gain of 5%; how much did he receive for it?

18. My salary is \$1500 a year; I spend 25% of it the first half year, and 35% the second half; how much do I save in a year? Ans. \$600.

Case II.

170. Given the percentage and rate, to find the base.

1. 60 is 5 per cent. of what number?

Solution.—If 5% of a number is 60, 1% of the number is $\frac{1}{5}$ of 60, or 12, and 100% of the number, which is the whole number, is 100 times 12, or 1200.

Since this is equivalent to multiplying by 100 and dividing by 5, and this last is equivalent to dividing by .05, we have the following

OPERATION.

$5\% = 60$
$100\% = 1200$ Ans.
or,
$60 \div .05 = 1200$

Rule.—*Divide the percentage by the rate expressed decimally.*

Note.—For young pupils the analysis will be simpler than the solution by the rule.

2. 45 is 20 per cent. of what number? Ans. 225.
3. 75 is 25 per cent. of what number? Ans. 300.
4. 96 is 20 per cent. of what number?
5. 230 is 5 per cent. of what number?
6. 112lb. is 40 per cent. of what weight?
7. 456 acres are 30 per cent. of how many?
8. 237 cows are 25 per cent. of how many? Ans. 948.
9. 157yd. are 12½ per cent. of how many? Ans. 1256.

10. 644 pigs are 35 per cent. of how many?

11. $78.18 is 33⅓ per cent. of how much?

Ans. $234.54.

12. A man spends $500 a year, which is 25% of his salary; what is his salary? Ans. $2000.

13. A has 280 acres of land, and this is 35% of what B has; how much has B?

14. A sold 36 pigs, which is 8% of what he now has; how many had he before the sale?

15. A boy found $15, which is 30% of what he then had; how much had he at first? Ans. $35.

16. A man had $13681.60 in a bank, and drew out 35% of it; how much did he draw out?

Ans. $4788.56.

Case III.

171. Given the base and percentage, to find the rate.

1. 25 is what per cent. of 125?

OPERATION.

$125 = 100\%$

$25 = \frac{25}{125} \times 100\%$

$= \frac{1}{5} \times 100\% = 20\%$

Solution.—125 is 100 per cent. of itself, and 25, which is $\frac{25}{125}$ of 125, is $\frac{25}{125}$ of 100 per cent., or $\frac{1}{5}$ of 100 per cent., which is 20 per cent. Hence the following

Rule.—*Take such a part of* 100 *as the percentage is of the base;* or, *multiply the percentage by* 100 *and divide by the base.*

2. 75 is what per cent. of 300? Ans. 25%.

3. 90 is what per cent. of 360? Ans. 25%.

4. 45 is what per cent. of 225? Ans. 20%.

5. 72 is what per cent. of 216? Ans. 33⅓%.

6. 96 is what per cent. of 128?

7. 48 is what per cent. of 120?

8. 112 is what per cent. of 896?

9. A man had $960, and lost $240; what per cent did he lose?

10. B lost $25, and then had $125; what per cent. of his money did he lose?

11. C sold 50 cows, which was 25 per cent. of the remainder; how many had he at first?

12. D gave his sister $480, and had $960 left; his money is now what per cent. of what he had at first?

FORMULAS.

172. The following formulas contain all the fundamental rules of percentage:

1. *Base × rate = percentage.*
2. *Percentage ÷ rate = base.*
3. *Percentage ÷ base = rate.*

TRADE DISCOUNTS.

173. A ***Trade Discount*** is a deduction from the price of goods, the amount of a bill, etc.

174. The ***List Price*** of goods is the fixed or published price. The ***Net Price*** is the list price minus the discount.

There are often several successive discounts deducted from a bill. Thus, when a bill is marked "20 and 10% off," it means that 20% is to be deducted from the bill and 10% from the remainder.

Sometimes one or more of the discounts are expressed as fractions; thus, "$\frac{1}{6}$ and 10% off."

1. A merchant sold a bill of goods amounting to $250, and marked it "10 and 5% off;" what was the net amount of the bill?

SOLUTION.—10% of $250 is $25, which taken from $250 leaves $225; and 5% of $225 is $11.25, which taken from $225 leaves $213.75.

2. A bill of hardware at list prices amounts to $264, and the discounts are "$\frac{1}{8}$ and 10% off;" what was the net amount of the bill? Ans. $207.90.

3. I sold a bill of valentines amounting to $68.50, and marked it "20, 10, and 5% off;" what did I receive for them? Ans. $46.85.

4. School-slates marked to sell at \$13.50 a case were sold at $\frac{1}{4}$, $\frac{1}{5}$, and 10% off; what was the net price?

Ans. \$7.29.

5. Bought a bill of envelopes amounting to \$75.20 on 30 days' credit at $\frac{1}{8}$ and 5% off, and 2% off for cash; what did I pay for them? Ans. \$61.26.

6. Bought a bill of valentines amounting to \$175.50 on 3 mo. credit at $\frac{1}{6}$, 10, and 5% off, and 2% for cash; what was the net cash amount of the bill? *Ans.* \$122.54.

7. How many per cent. off a bill is 10 and 10% off? Is 10 and 5% off? Is 15 and 5% off?

Ans. 19%; $14\frac{1}{2}$%; $19\frac{1}{4}$%.

8. What is the difference between 15 and 10% off and 10 and 15% off? Between 10 and 5% off and 5 and 10% off?

BONDS AND INCOME.

175. ***Bonds*** are written or printed obligations to pay a certain sum of money at or before a specified time.

176. These bonds pay the holders a certain income or interest at definite periods—annually, semi-annually, or quarterly—at some definite rate per cent. per year.

177. ***Coupon Bonds*** have attached to them "coupons," or certificates for the interest as it becomes due. These coupons are to be cut off, and the interest is paid to the bearer.

178. ***Registered Bonds*** have no coupons attached, and the interest is payable only to the bondholder, whose name is registered in the books of the company or organization issuing the bonds.

Bonds are often named from the yearly percentage they pay, as 6s, 5s, $4\frac{1}{2}$s, etc. When bonds are very secure, they often sell for more than the face value, and are said to be at a *premium.* When they sell for less than their face value, they are said to be at a *discount.*

CASE I.

179. To find the income obtained from bonds.

1. What is the annual income derived from \$2000 6% State bonds?

SOLUTION.—The income on \$100 is \$6, and on \$2000, which is 20 times \$100, the income is 20 times \$6, or \$120.

RULE.—*Multiply the income on \$100 by the number of hundred dollars on the face value of the bonds.*

2. What is the annual income on \$1200 5% State bonds of \$100 each? Ans. \$60.

3. I have \$7000 Philadelphia 6's, interest payable semi-annually; what is the semi-annual income? Ans. \$210.

4. I bought 76 4½% \$100 government bonds, payable quarterly; what is my income from them every three months? Ans. \$85.50.

CASE II.

180. To find the cost of bonds when at a premium or discount.

1. What must I pay for 20 U. S. 4% \$100 bonds quoted at 123½?

SOLUTION.—They will cost 20 times \$123½, or \$2470.

RULE.—*Multiply the price per \$100 by the number of hundreds bought or sold.*

2. What must I pay for 25 Lehigh Valley 6% \$100 bonds quoted at 125¼? Ans \$3131.25.

3. What must I pay for 36 Reading R. R. 7's (\$100) quoted at 128¾? Ans. \$4635.

4. I bought \$8600 U. S. 4% bonds at 106¾; what did they cost me? Ans. \$9180.50.

5. I bought 30 St. Louis 7% bonds at 96¼, and sold them at 110½; what did I gain by the transaction? Ans. \$427.50.

STOCKS AND DIVIDENDS.

181. The ***Stock*** of a company represents the money invested in its business. It is divided into equal parts, called *shares.*

182. ***A Dividend*** is a sum to be paid to the stockholders out of the gains of the company. It is divided in proportion to the number of shares held by them.

183. The ***Par Value*** of stock is its nominal value as fixed by the charter, or articles of agreement, of the company. It is usually $50 or $100 per share; although other sums are often agreed upon.

184. The ***Real Value*** of stock is what it will sell for.

185. ***Premium*** is how much its real value *exceeds* its par value.

186. ***Discount*** is how much its real value *is less* than its par value.

CASE I.

187. Given the stock and rate of dividend, to find the dividend.

1. A owns 50 shares of bank-stock, at $100 each; the bank declares a dividend of 6%; required A's dividend.

SOLUTION.—If one share is worth $100, 50 shares are worth 50 times $100, or $5000. Hence A's stock is worth $5000. His dividend is .06 times $5000, or $300.

OPERATION.

$$\begin{array}{r} 50 \\ \underline{100} \\ 5000 \\ \underline{.06} \\ \$300.00 \end{array}$$

RULE.—*Multiply the par value of the number of shares by the rate of dividend.*

NOTE.—Or, multiply the amount of dividend on one share by the number of shares.

2. A man owns 56 shares of stock, at $50 per share

par value; the company declares a 5% dividend; what is his dividend? Ans. $140.

3. A lady has 175 shares of stock, at $10 par per share; the company declares 8½% dividend; what is her dividend? Ans. $148.75.

4. I hold 250 shares of mining stock, at $20 par a share; the company has divided 5½%; what is my dividend? Ans. $275.

Case II.

188. Given the par value and rate of premium or discount, to find the premium, discount, or real value.

1. A bought 25 shares of stock ($50), at 4% premium; required the premium and the real value.

OPERATION.

$$\begin{array}{rl} \$50 \times 25 = \$1250 & = \text{par value.} \\ .04 & \\ \hline 50.00 & = \text{premium.} \\ 1250. & \\ \hline \$1300 & = \text{real value.} \end{array}$$

Solution.—The par value of 25 shares at $50 a share is 25 times $50, or $1250. The premium is .04 times $1250, or $50, and the premium added to the par value equals $1300, the real value.

Rule.—*Multiply the par value by the rate, for amount of premium or discount. Add or subtract this from par value, for real value.*

2. I bought 75 shares of gas stock ($20), at 3½% premium; required the premium and the cost. Ans. $52.50; $1552.50.

3. I sold 120 shares of stock ($15), at 5% discount; required the discount, and the amount received for it. Ans. $90; $1710.

4. Sold 34 shares of R. R. stock ($50), at 2½% discount; what was received for it? Ans. $1657.50.

5. Bought 12 shares bank stock ($50), at 12½% premium, and afterwards sold the same at $60 per share; did I gain or lose, and how much? Ans. Gained $45.

189. ***Commission*** is a percentage paid to an agent for the transaction of business.

190. Such agents are known by various names, as *commission merchants, brokers, correspondents,* etc.

191. A *Stock Broker* is a person who buys and sells stocks, bonds, etc. for others.

192. *Commission* is *estimated* at a certain rate per cent. on the *actual amount* of the *sale, purchase, collection,* or *exchange.*

193. *Brokerage* is *estimated* on the *par value* of the stocks or bonds bought or sold.

194. The *rate of brokerage* is usually $\frac{1}{4}\%$ on face value, and will be so understood when no other rate is named. In New York it is $\frac{1}{8}\%$ on both bonds and stocks.

Stocks are quoted either at the price of one share, or at the price of \$100 of par value of the stock. The former method is used in Philadelphia, the latter in New York.

Thus, stock whose par value is \$50 a share, and is selling at \$35½ a share, is quoted at 35½ in Philadelphia and at 71 in New York.

Notice that when the brokerage is $\frac{1}{4}\%$, it is \$⅛ on a share of \$50 par, and when brokerage is $\frac{1}{8}\%$, it is \$$\frac{1}{16}$ on a share of \$50 par.

Case I.

195. Given the base and the rate, to find the commission or brokerage.

1. An agent bought some goods for \$580, his rate of commission being $2\frac{1}{2}\%$; what was the commission?

	OPERATION.
SOLUTION.—The commission was .02½ times \$560, which we find by multiplying is \$14.50.	\$580 .02½ \$14.50

RULE.—*Multiply the base by the rate to find the commission or brokerage.*

2. I bought through a commission merchant $750 worth of dry goods, commission $3\frac{1}{2}$%; what did they cost me?
Ans. $776.25.

3. A broker bought for me 25 shares of bank stock at $56 a share, par value $50 a share, brokerage $\frac{1}{4}$%; required the brokerage and the cost of the stock.
Ans. $1403.12½.

4. A broker bought for me 46 shares of Penna. railroad stock ($50) at $54\frac{1}{2}$, brokerage $\frac{1}{4}$%; what did the stock cost me?
Ans. $2512.75.

5. Mr. Smith sells a property through an agent for $7650, paying the agent $3\frac{1}{4}$%; what does Mr. Smith receive for the property?
Ans. $7401.37½.

6. My agent bought 20 horses for $130 each, and paid $75 expenses for transportation; what did they cost me, his commission being $2\frac{3}{4}$%?
Ans. $2746.50.

7. My broker bought on my account 40 shares of bank stock ($100) at $108\frac{1}{2}$, and sold it a few days after for $114\frac{1}{4}$; what did I clear, brokerage $\frac{1}{8}$% on both buying and selling?
Ans. $220.

CASE II.

196. Given the cost or proceeds or commission and the rate, to find the base.

1. An agent received $2050 to be invested in land after retaining $2\frac{1}{2}$% on the purchase for commission; required the cost of the land.

SOLUTION.—The *sum* invested, increased by $2\frac{1}{2}$% of itself, equals $1.02\frac{1}{2}$ times the *sum*; if $1.02\frac{1}{2}$ times the *sum* equals $2050, the *sum* equals $2050 divided by $1.02\frac{1}{2}$, which we find is $2000.

OPERATION.

$$\frac{2050}{1.02\frac{1}{2}} = 2000$$

RULE I.—*Divide the amount of commission or brokerage by the rate to find the base.*

RULE II.—*Divide the entire cost by 1 plus the rate, or the net proceeds by 1 minus the rate, to find the base.*

NOTE.—In stock brokerage, *divide the net proceeds by the market value minus the rate, or the entire cost by the market value plus the rate, to find par value.*

2. An agent received $12 commission for buying goods at the rate of $2\frac{1}{2}\%$; what was the cost of the goods? Ans. $480.

3. An agent's commission for collecting money at $3\frac{1}{4}\%$ is $18.20; how much money did he collect for his employer? Ans. $560.

4. A broker received $6.40 for selling bonds at the rate of $\frac{1}{4}\%$; what amount of bonds did he sell? Ans. $2560.

5. A broker received $1738 to invest in stock ($50), deducting $\frac{1}{8}\%$ for brokerage; how many shares did he buy, the stock selling at $54\frac{1}{4}$? Ans. 32 shares.

INSURANCE.

197. ***Insurance*** is security against loss by fire, water, accident, etc. It is of two kinds—***Property Insurance*** and ***Life Insurance.***

198. ***Property Insurance*** is a contract for the payment of a sum of money for the loss of property by fire, transportation, etc.

199. ***Life Insurance*** is a contract for the payment of a specified sum at the death of a person, or to the person at a certain age.

200. The ***Policy*** is the written agreement between the insurer and the insured.

201. The ***Premium*** is the sum paid for insurance. It is a certain rate per cent. of the amount insured.

202. The party or company insuring is called an ***underwriter,*** because the name is written under the policy..

CASE I.

203. Given either two of the three quantities, value, rate, or premium, to find the third.

1. A man insured his house for \$6750 at $1\frac{1}{2}\%$; required the premium.

	OPERATION.
SOLUTION.—The premium on \$6750 at $1\frac{1}{2}\%$ is $.01\frac{1}{2}$ times \$6750, which we find is \$101.25.	\$6750 $.01\frac{1}{2}$ \$101.25

RULE.—*Multiply the amount insured by the rate, to find the premium.*

NOTES.—1. To find the *value* of the property, *divide the premium by the rate.*

2. To find the *rate of insurance, divide the premium by the value.*

2. A man insured his store for 3 years for \$4650 at $1\frac{1}{2}\%$; required the premium. Ans. \$69.75.

3. What is the premium for an insurance of \$2500 on my furniture for 6 years at $1\frac{1}{4}\%$? Ans. \$31.25.

4. I insured my house for \$4000, my furniture for \$1500, and my library for \$500, at $1\frac{1}{8}\%$; what was the cost of insurance? Ans. \$67.50.

5. I paid \$45 to insure the transportation of goods at $\frac{3}{4}\%$; what sum was insured on the goods? Ans. \$6000.

6. The premium for insuring $\frac{3}{4}$ of the value of a house for 3 years at $\frac{3}{4}\%$ was \$45; what was the value of the house? Ans. \$8000.

7. A man effected an insurance of \$4800 on his life at the rate of \$15 a year on \$1000; if he dies in 20 yrs., how much will his heirs receive more than he paid for insurance? Ans. \$3360.

8. A man effected an insurance of \$7500 on a factory and its machinery for 5 years; if the premium was \$243.75, what was the rate of insurance? Ans. $3\frac{1}{4}\%$.

SIMPLE INTEREST.

204. ***Interest*** is money charged for the use of money. It is estimated at a certain rate per cent. *per annum.*

Thus, if Mr. Smith borrowed \$2000 of Mr. Jones, Mr. Jones would charge Mr. Smith some rate per cent., say 6%, or \$6 on \$100, for a year for the use of the money.

205. The ***Principal*** is the sum for the use of which interest is charged.

206. The ***Rate of Interest*** is the rate per cent. charged for one year or some other specified time.

207. The ***Time*** is how long the money is on interest.

208. The ***Amount*** is the sum of the principal and interest.

209. ***Simple Interest*** is interest upon the principal only. ***Compound Interest*** is interest upon the principal and interest.

210. ***Legal Interest*** is the rate established by law. ***Usury*** is a rate greater than the legal rate. The taking of usury is prohibited by law.

211. The quantities considered in Simple Interest are the *Principal, Rate, Time, Interest,* and *Amount.*

In computing interest it is customary to consider 30 da. a month and 12 mo. a year.

CASE I.

212. Given the principal, rate per cent., and time, to find the interest or amount.

Method by Years.

1. What is the interest of \$2400 for 6yr. 7mo. 15da. at 7%?

SOLUTION.—By reduction we find that 6yr. 7mo. 15da. equal $6\frac{5}{8}$yr. The interest of \$2400 for 1yr. is .07 times 2400, which is \$168; and for $6\frac{5}{8}$yr. it is $6\frac{5}{8}$ times \$168, which by multiplying we find is \$1113. Hence the following rule:

OPERATION.

\$2400
.07
168.00
$6\frac{5}{8}$
\$1113.00 Ans.

RULE.—*Multiply the principal by the rate per cent., expressed decimally, and that product by the time expressed in years.*

NOTE.—This method is practical when the months and days are a simple fractional part of a year.

EXAMPLES FOR PRACTICE.

Required the interest

2. Of \$180 for 3yr. 6mo. at 7%. Ans. \$44.10.
3. Of \$470 for 7yr. 8mo. at 6%. Ans. \$216.20.
4. Of \$172 for 5yr. 9mo. at 5%. Ans. \$49.45.
5. Of \$480 for 5yr. 10mo. at 12%.
6. Of \$1080 for 3yr. 7mo. 6da. at 5%.
7. Of \$1260 for 2yr. 2mo. 12da. at 5%.
8. Of \$1000 for 3yr. 8mo. 12da. at 10%.

Six Per Cent. Method.

213. The *six per cent. method* is so called because the method is based on that rate.

1. What is the interest of \$240 for 2yr. 8mo. 12da. at 6%?

SOLUTION.—2yr. 8mo. equal 32mo. The interest of \$1 for 12mo. is 6cts., and for 1mo. it is $\frac{1}{12}$ of 6cts., or $\frac{1}{2}$ct., and for 32mo. it is $32 \times \frac{1}{2}$ct. = 16cts.

Since the interest on \$1 for 1mo., or 30da., is $\frac{1}{2}$ct., or 5 mills, for 1da. it is $\frac{1}{30}$ of 5 mills, or $\frac{1}{6}$ of a mill, and for 12da. it is $12 \times \frac{1}{6}$ mills = 2 mills.

OPERATION.

2yr. 8mo. = 32mo.

$$
\begin{array}{r}
32 \times \frac{1}{2} = \$0.16 \\
12 \times \frac{1}{6} = \underline{\quad .002} \\
\$0.162 \\
\underline{240} \\
\$38.88
\end{array}
$$

Hence the interest on \$1 for 32mo. and 12da. is 16cts. plus 2 mills, or \$0.162. If the interest on \$1 is \$0.162, on \$240 it is 240 times \$0.162, which equals \$38.88. From this we have the following

RULE.—1. *Take* ONE-HALF *of the number of months as cents, and* ONE-SIXTH *of the number of days as mills; their sum will be the interest of one dollar, for the given time, at 6 per cent.*

2. *Multiply this by the principal, and the product will be the interest at 6 per cent. For any other rate, take as many sixths of it as that rate is of six.*

NOTE.—1. For 7% add $\frac{1}{6}$, for 8% add $\frac{1}{3}$, for 9% add $\frac{1}{2}$, for 5% subtract $\frac{1}{6}$, for 4% subtract $\frac{1}{3}$, etc.

2. When the time is brief, the rule of business men is as follows: "*Multiply dollars by days, and divide by* 6000."

EXAMPLES FOR PRACTICE.

Required the interest of

2. $480 for 3yr. 8mo. 18da. at 6%. Ans. $107.04.
3. $256 for 7yr. 4mo. 24da. at 6%. Ans. $113.66.
4. $48.25 for 3yr. 6mo. 6da. at 6%. Ans. $10.18.
5. $50.50 for 6yr. 10mo. 18da. at 7%. Ans. $24.33.
6. $28.25 for 5yr. 7mo. 24da. at 5%.
7. What is the amount of $360 for 2yr. and 6mo. at 6 per cent.? Ans. $414.
8. What is the amount of $250 for 3yr. 8mo. 18da. at 6 per cent.? Ans. $305.75.
9. What is the amount of $620 for 5yr. 10mo. 24da. at 7 per cent.? Ans. $876.06.
10. Mary's father put out $500 on interest, at 10%, at her birth; how much will she be worth when she is 21 years of age?
11. Required the difference between the interest and the amount of $624 for 3yr. 8mo. 15da., at 5 per cent.

NOTE.—For other exercises under this rule solve the problems in the previous and following cases.

The 60 Day Method.

214. At 6% a year the rate for 2mo., or 60da., is 1%; hence, *for 60 days* $\frac{1}{100}$ *of the principal equals the interest.*

215. From this principle, by taking aliquot parts of 60, we can readily find the interest for any number of days. Hence the following

RULE.—*Point off two places in the principal to find the*

interest for 60 days, and take multiples or aliquot parts of this interest for any other number of days.

NOTE.—In most problems in interest the 60 day method requires the least work.

1. What is the interest of \$240 at 6% for 66 days? For 96 days?

SOLUTION.—We point off two places to find the Int. for 60 da.; then take $\frac{1}{10}$ of this Int. to find the Int. for 6da.; the sum of the two will be the Int. for 66da.

OPERATION.

\$2.40 =	Int.	for	60da.
.24 =	"	"	6da.
\$2.64 =	"	"	66da.

SOLUTION.—We point off two places with a vertical line, to find the Int. for 60 da.; take $\frac{1}{2}$ of \$2.40 to find the Int. for 30da., and take $\frac{1}{5}$ of \$1.20 to find the Int. for 6da.; the sum of the three gives the Int. for 96da. \$3.84.

OPERATION.

\$2	40 =	Int.	for	60da.
1	20 =	"	"	30da.
	24 =	"	"	6da.
\$3.84 =		"	"	96da.

NOTE.—In practice it is convenient to draw a vertical line between dollars and cents, as in the second solution.

2. Find the Int. of \$360 at 6% for 6mo. 12da. For 128 days.

OPERATION.

\$3	60 =	Int.	for	60da.
10	80 =	"	"	6mo. (3 × 2mo.)
	60 =	"	"	10da. ($\frac{1}{6}$ of 60da.)
	12 =	"	"	2da. ($\frac{1}{5}$ of 10da.)
\$11.52 =		"	"	6mo. 12da.

OPERATION.

\$3	60 =	Int.	for	60da.
7	20 =	"	"	120da.
	36 =	"	"	6da.
	12 =	"	"	2da.
\$7.68 =		"	"	128da.

Find the Int.

3. Of \$240 for 93da. at 6%. Ans. \$3.72.
4. Of \$450 for 100da. at 6%. Ans. \$7.50.
5. Of \$270 for 118da. at 6%. Ans. \$5.31.
6. Of \$180 for 8mo. 15da. at 6%. Ans. \$7.65.
7. Of \$135 for 4mo. 18da. at 6%. Ans. \$3.10$\frac{1}{2}$.
8. Of \$780 for 4yr. 8mo. at 6%. Ans. \$218.40.
9. Of \$450.25 for 1yr. 7mo. 25da. at 6%. Ans. \$44.65.
10. Of \$1260 for 3yr. 6mo. 15da. at 8%. Ans. \$357.

NOTE.—Pupils may solve by this method the problems under the previous methods.

PROMISSORY NOTES.

216. A ***Promissory Note*** is a written agreement to pay a certain sum of money on demand or at a specified time.

217. The sum of money specified is called the ***Face*** of the note.

218. The person who signs the note is called its ***Maker.*** The person to whom it is payable is called the ***Payee.***

219. The following is the usual form of a note:

$$240\frac{25}{100}$. *Philadelphia, Mar. 20, 1887.*

Three months after date I promise to pay to Henry Wilson, or order, Two Hundred and Forty $\frac{25}{100}$ Dollars, with interest at 6%, value received, without defalcation.

John Smith.

In the above note the *face* is $\$240\frac{25}{100}$; the *maker* is John Smith; the *payee* is Henry Wilson.

If Henry Wilson wishes to transfer this note to Thomas Arnold, he will write on its back the words, "Pay to the order of Thomas Arnold," and sign his name. Henry Wilson is then called the *indorser*, and Thomas Arnold the *holder*.

A note should contain the words "value received," otherwise the holder may be required to prove that value was received. The words "without defalcation" are required in Pennsylvania to make a note negotiable, and in New Jersey "without defalcation or discount."

It is the custom to allow a note to run *three days* beyond the specified day of payment. These three days are called *days of grace*, and are usually included in reckoning the interest.

1. Find the interest on a 3-months note for $600, at 6%, dated Jan. 16, 1887.

		OPERATION.
15da. in Jan.		
28 " " Feb.	$6|00 =	Int. for 60da.
31 " " Mar.	3|00 =	" " 30da.
16 " " Apr.	|30 =	" " 3da.
90da.	$9.30 =	" " 93da.

SOLUTION.—The exact number of days from Jan. 16 to April 16 is 90da.; adding 3 days of grace gives 93 days; computing the interest by the third method, we have $9.30.

Rule.—*To compute the interest on short-time notes, find the exact time in days, add 3 days of grace, and compute the interest for this time.*

Note.—When interest on a note is payable annually, the interest for a year is readily found by multiplying by the rate.

Find the interest on the following short-time notes:

2. Dated May 12, due in 3mo., for \$480, at 6%.
Ans. \$7.60.
3. Dated June 10, due in 4mo., for \$600, at 5%.
Ans. \$10.42.
4. Dated July 15, due in 4mo., for \$840, at 7%.
Ans. \$20.58.
5. Dated Nov. 20, due in 4mo., for \$960, at $6\frac{1}{2}$%.
Ans. \$21.32.
6. Dated Dec. 18, 1887, due in 3mo., for \$1200, at $5\frac{1}{2}$%.
Ans. \$17.23.

Note.—Let the pupil put each of the above problems in the form of a note.

DISCOUNTING NOTES.

220. ***Discount*** is an allowance made for the payment of a note before it is due.

221. Discount is reckoned by taking the interest on the face of the note for the time from the day it is discounted to the date of its payment.

222. The sum received for the note is called the *proceeds.* The proceeds equal the face of the note less the discount.

To illustrate, suppose a person wishes to borrow some money for a short time—say \$500 for 60 days. He would take a note for \$500, at 6%, made by himself or some other party, to a bank; and if properly indorsed, the bank will take the note and pay him the face of the note less the interest for the time (plus 3 days) the note has to run.

223. In such cases the note is usually drawn *without interest.* When a note *drawing interest* is discounted, the discount is reckoned on the *amount* of the note when it becomes due.

In some States both the *day of discount* and the *day of payment* are included in computing the time, which amounts to adding *four* days of grace. The problems given below are solved by adding *three days.*

1. A bank discounted a note for $400, drawn for 60 days, at 6%; find the discount and proceeds.

SOLUTION.—The time equals 60da. + 3d. = 63da. The interest of $400 for 63 days at 6% is $4.20. The proceeds equal $400 minus $4.20, or $395.80.

OPERATION.

4|00 =	Int. for 60da.
|20 =	" " 3da.
4.20 =	" " 63da.

$400 — $4.20 = $395.80.

RULE.—1. *To find the discount, find the interest of the sum discounted for the specified time plus three days.*

2. *Subtract the discount from the face, to find the proceeds.*

NOTE.—The discount of an *interest-bearing* note is computed on the *amount* of the note at its maturity.

EXAMPLES FOR PRACTICE.

2. Find the bank discount of a note for $250, due in 30 days, discounted at 6%. Ans. $1.37½.

3. Find the bank discount of a note for $600, due in 60 days, discounted at 6%. Ans. $6.30.

4. Find the proceeds of a note for $800, due in 90 days, discounted at 8%. Ans. $783.47.

5. A note for $900, due in 3mo., was discounted at 7%; required the proceeds. Ans. $883.72½.

6. What are the proceeds of a note of $360, dated May 20 and payable Aug. 16, discounted at 6%? Ans. $5.40.

7. A note of $600, dated July 12 and payable Nov. 22, was discounted at 8%; what were the proceeds? Ans. $581.87.

8. A man sold on Dec. 24 a note of $480, dated Oct. 12, 1887, and due Mar. 20, 1888, at 6% discount; what did he receive for it? Ans. $472.80.

NOTE.—The time of discount is reckoned from the date of discount, Oct. 12. Notice also that February contains 29 days.

9. Holding a note for $2400, dated Jan. 20, 1888, and due May 10, 1888, I had it discounted on Feb. 16 for 8%; what did I receive for it? Ans. $2353.60.

10. Find the discount and proceeds of the following note:

560\frac{50}{100}$. *New York, May 24, 1888.*

Three months after date I promise to pay Thomas Dolan, or order, Five Hundred and Sixty $\frac{50}{100}$ Dollars, value received.

Cecil Harper.

Discounted June 16, 1888, at 6%.

Ans. Discount, $6.73; proceeds, $553.77.

NOTE.—This note is due Aug. 24, or three days after, on the 27th; hence, if discounted June 16, the time for which it is discounted is 72 days.

11. Find the discount and proceeds of the following note:

$360. *Chicago, July 20, 1888.*

Four months after date I promise to pay to Jonas Martin, or order, Three Hundred and Sixty Dollars, value received.

Henry Bowman.

Discounted Sept. 10, 1888, at 6%.

Ans. Discount, $4.44; proceeds, $355.56.

NOTE.—This note is due Nov. 20, or, adding 3da. grace, Nov. 23; hence, if discounted Sept. 10, the time of discount is 74 days.

12. Find the discount and proceeds of the following note:

$600. *Philadelphia, Nov. 16, 1887.*

Four months after date I promise to pay to Mary Smith, or order, Six Hundred Dollars, with interest at 6%, value received, without defalcation. *John Smith.*

Discounted Jan. 10, 1888, at 6%.

Ans. Discount, $7.14; proceeds, $604.26.

NOTE.—The time on interest, including 3da. grace, is 124 days. The *amount* is $612.40. The time of discount, from Jan. 10 to Mar. 16, is 66da.; and we add 4 days, as the custom in Pennsylvania is to include the day of discount and the day of payment, which makes 70da. The Int. on $612.40 for 70da. is $7.14, etc.

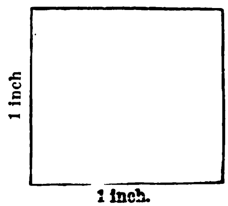
SECTION IX.

PRACTICAL MEASUREMENTS.

MEASURES OF SURFACES.

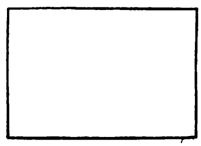
224. The ***Area*** of a surface is the surface included within its sides. The area is expressed by the number of times it contains a small square as the *unit of measure.*

225. A ***Square*** is a plane surface which has four equal sides and four right angles, as in the margin.

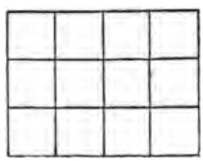


226. A ***Rectangle*** is a plane surface which has four sides and four right angles, as a door, a slate, a pane of glass, etc.

227. There are as *many squares* in a *rectangle* as the *number of units in the length multiplied by the number of units in the width.*



For, suppose this rectangle to be 4 units long and 3 units wide; then, if we divide it into little squares, we see that there are 4 squares in one row; and there are 3 rows: hence, there are 3 times 4, or 12, squares in all, and this is the same as the number of units in length multiplied by the number in width.



NOTE.—The teacher will illustrate these areas on the board until the pupil has a clear idea of the subject.

RULE.—*To find the area of a square or rectangle, multiply the length by the width.*

NOTE.—To find *either side* of a rectangle, *divide the area by the other side.*

1. How many square feet in a floor 24 feet long and 20 feet wide? How many square yards?

OPERATION.

$$\begin{array}{r} 24 \\ \underline{20} \\ 9)\underline{480}\text{sq. ft.} \\ 53\tfrac{1}{3}\text{sq. ft.} \end{array}$$

SOLUTION.—The area equals 24×20, or 480sq. ft.; dividing by 9 to reduce to square yards, we have $53\frac{1}{3}$sq. yd.

EXAMPLES FOR PRACTICE.

2. How much writing-surface in a school-slate 10in. long by $7\frac{1}{2}$in. wide inside the frame? Ans. 75sq. in.

3. How many square feet in the top of a table 6ft. long and 3 ft. wide? Ans. 18ft.

4. What is the area of the floor of a room 26ft. long and $12\frac{1}{2}$ft. wide? Ans. 325sq. ft.

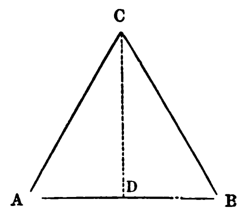
5. What is the width of my slate if it is 12in. long and contains $\frac{2}{3}$ of a square foot? Ans. 8in.

6. How many sq. yds. in the four sides of a room which is 24ft. long, 16ft. wide, and 12ft. high? Ans $106\frac{2}{3}$sq. yds.

7. What is the entire surface of a cubical box each of whose sides is 2 feet? Ans. 24sq. ft.

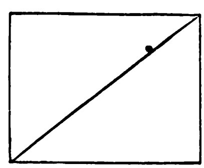
8. Measure the parlor at your home, and find how many square yards of paper will cover the walls and ceiling.

THE TRIANGLE.



228. A ***Triangle*** is a plane figure having three sides and three angles, as A B C.

229. The side A B, on which it seems to stand, is called the *Base*. The line C D perpendicular to the base is the *Altitude*.



230. If you take a rectangle and cut it into two parts by cutting from corner to corner, you will find that each triangle is equal to one-half of the rectangle. Hence the following

RULE.—*To find the area of a triangle, multiply the base by one-half of the altitude.*

NOTE.—To find the *base* or the *altitude* of a triangle, *divide the area by one-half of the other dimension.*

1. What is the area of a triangle whose base is 12in. and altitude 8in.?

SOLUTION.—To find the area of a triangle, we multiply the base by one-half the altitude; hence the area $= 12 \times 4 = 48$ sq. in.

EXAMPLES FOR PRACTICE.

2. How many square yards in a triangular grass-plat whose base is 42ft. and altitude 15ft.? Ans. 35sq. yd.

3. How many sq. yds. in a triangular bed of flowers whose base is $19\frac{1}{2}$ft. and altitude $13\frac{1}{2}$ft.? Ans. $14\frac{5}{8}$sq. yd.

4. The area of a triangular lot is 560sq. yd., and the base is 120 feet; what is the altitude? Ans. 84ft.

5. How many square feet of boards will it take to cover the two gable-ends of a house in the form of a triangle, the width of the house being 30ft. and the height of the ridge 12ft.? Ans. 360sq. ft.

MEASUREMENT OF LAND.

231. Land is usually measured by the *acre.* Small portions of land are estimated in *perches* or *square rods.*

232. Pupils will remember that 160 *square rods*—called also *perches*—equal 1 *acre,* and that a *square rod* is $16\frac{1}{2}$ feet on a side.

Remember also that if we "*multiply rods by rods*" the product is *square rods.*

Let the pupils mark out a *square rod* in the playground, and then think that it takes 160 of these to make an acre. They may, in the country, measure off a square 209ft. on a side, and this square will be very nearly one acre.

1. How many acres in a rectangular field 40 rods long by 16 rods wide?

SOLUTION.—The area equals $40 \times 16 = 640$ *square rods,* which, dividing by 160, equals 4 acres.

EXAMPLES FOR PRACTICE.

2. How many acres in a rectangular field 80 rods long by 20 rods wide? Ans. 10 acres.

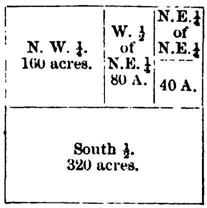
3. How many perches in a rectangular building-lot 300 feet long and 66 feet wide? Ans. $72\frac{8}{11}$ perches.

4. What cost a farm in the form of a rectangle, 120 rods long by 84 rods wide, at $80 an acre? Ans. $5040.

5. Mr. Brown has a field in the form of a rectangle, 64 rods long, containing 24 acres; what is its width?

Ans. 60 rods.

The townships in the West, as laid out by the U. S. Government, are squares 6 miles on a side, containing 36sq. miles, and each square mile is called a *section*. Each section (640 acres) is divided into *half-sections, quarter-sections, half-quarter-sections*, etc. The method of division and reading is shown in the diagram.



1 section = 640 acres.

PLASTERING, PAINTING, PAVING, ETC.

233. ***Plastering, Painting,*** etc., are usually estimated by the *square yard*. There is no uniform rule for the allowance to be made for doors and windows.

234. ***Roofing, Flooring, Tiling,*** and ***Paving*** are usually estimated by *the square*, which equals 100 *square feet*. They are also estimated by the *square foot* and *square yard*.

The pupils will notice, if they have an opportunity, how plastering, flooring, paving, etc., are done, and by inquiring of those who are doing the work may learn something more in respect to these things.

1. What will be the expense of paving a sidewalk 360ft. long and 8ft. wide, at $0.50 a square yard?

SOLUTION.—The area equals $360 \times 8 = 2880$ sq. ft., which, dividing by 9, equals 320sq. yds. The cost is $\$0.50 \times 320 = \$160$.

EXAMPLES FOR PRACTICE.

2. What will the lumber cost to floor a room 24ft. long by 16ft. wide, at 48¢ a square yard? Ans. $20.48.

3. What will it cost to paint the ceiling of a room 30ft. by 15ft., at 15¢ a square yard? Ans. $7.50.

4. What will it cost to paint a front-yard fence 66ft. long and 3ft. high, at 25cts. a square yard? Ans. $5.50.

5. What will it cost to wainscot a music-room which is 27ft. long, 18ft. wide, to a height of 4ft. 6in., at $1.50 per square yard? Ans. $67.50.

6. What will it cost to plaster a school-room 42ft. long, 25ft. wide, and 15ft. high, at 12¢ a square foot, an allowance of 20sq. yd. being made for doors and windows?

Ans. $38.40.

CARPETING, PAPERING, ETC.

235. In ***Carpeting*** we take into consideration the width of the carpet, the allowance for matching the figures, and whether the strips run lengthwise or across the floor.

236. To match the figures, we must often turn under or cut off one of the ends. Also an exact number of strips, or breadths, may be a little too wide for the room, and must be turned under.

The pupils will notice how the carpets are laid in the different rooms at home, and estimate the number of yards required.

RULE.—*To find the number of yards of carpet to cover a floor, find the number of strips required, and multiply the number of yards in each strip by the number of strips.*

NOTE.—A similar rule applies to finding the amount of paper required to paper a room. Wall-paper is usually 18in. wide.

1. How many yards of carpet, 1 yard wide, will it take to carpet a floor 24ft. long by 15ft. wide, no allowance being made for matching?

SOLUTION.—Each strip of carpet will be 24ft. long; and since the room is 15ft., or 5yds., wide, and the carpet is 1yd., it will require 5 strips; hence it will take 24ft. $\times$ 5, or 120ft., or 120 $\div$ 3 $=$ 40yd.

EXAMPLES FOR PRACTICE.

2. What will it cost to put oilcloth, at 65¢ a yard, on a kitchen floor 15ft. by 12ft., no allowance being made for waste or matching? Ans. $13.

3. A lady wishes to cover her sitting-room, 31ft. long by 15ft. wide, with matting $1\frac{1}{2}$ft. wide, no allowance required for matching; how many yards will it take running lengthwise? How many running crosswise? Ans. $103\frac{1}{3}$yd.; 105yd.

4. How many yards of Brussels carpet, $\frac{3}{4}$yd. wide, will it take to carpet a parlor 24ft. long by 15ft. wide, the strips running lengthwise and the matching of figures requiring 8in. to be cut off each strip except the first? Ans. $57\frac{1}{3}$.

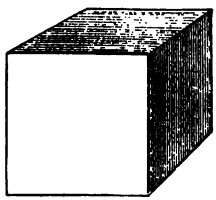
5. How many yards of paper, 18in. wide, will it take to paper a room 30ft. long, 20ft. wide, and 12ft. high, no deduction being made for doors and windows?

SOLUTION.—The length around the room is $(30 + 20) \times 2 = 100$ft., or 1200 inches, and it will take $1200 \div 18$, or about 67, pieces 12ft. in length, or $67 \times 12 \div 3 = 268$ yards, to cover the walls of the room.

NOTE.—Usually no deduction is made for doors and windows, this space being counted to cover waste in matching, etc.

MEASURE OF VOLUME.

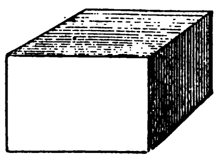
237. A ***Solid*** or ***Volume*** is anything which has length, breadth, and thickness, as a box, a block of marble, etc.



1 inch.

238. A ***Cube*** is a solid bounded by six equal squares. Its length, breadth, and thickness are equal.

239. A ***Rectangular Solid*** is a solid bounded by six rectangles. The bounding rectangles are called *faces*. Cellars, boxes, rooms, etc., are examples of rectangular solids.

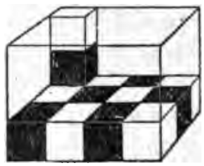


A Rectangular Solid.

240. By the ***Contents*** of a cube or rectangular solid we mean the amount of space or material it con-

tains. The contents are expressed by the number of times the solid contains a cube as a *unit of measure.*

241. There are as many cubes in a rectangular solid as *the product of the number of units in the length, breadth, and height.*



For, in the base of the solid there are 4 little cubes in a row and 3 rows, hence there are in the base 4×3, or 12, little cubes; and there are three layers of these cubes: hence in all there are $12 \times 3 = 36$ little cubes, and this is equal to the number of units in the length of the solid multiplied by the number in the breadth, and that product multiplied by the number in the height.

NOTE.—If the teacher has a number of little cubes, he can illustrate this very clearly.

RULE.—*To find the contents of a rectangular solid, take the product of its length, breadth, and height.*

1. How many cubic feet in a rectangular block of marble 8ft. long, 4ft. wide, and 3ft. high?

SOLUTION.—To find the contents, we take the product of the length, breadth, and height, and we have $8 \times 4 \times 3 = 96$cu. ft.

EXAMPLES FOR PRACTICE.

2. What are the contents of a room 18ft. long, 14ft. wide, and 10ft. high? Ans. 2520cu. ft.

3. A cistern is 6ft. square and 9ft. deep; how many cubic feet will it hold? Cubic yards? Ans. 12cu. yd.

4. How many cubic yards of air in a room 24ft. long, 16ft. wide, and 12ft. high? Ans. $170\frac{2}{3}$cu. yd.

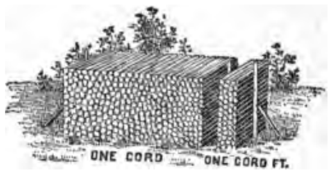
5. How many cubic yards of earth were dug from a cellar 48ft. long, 36ft. wide, and 7ft. 3in. deep? Ans. 464cu. yd.

6. How many gallons of water will a cistern hold which is 8 ft. square and 7ft. deep, a gallon being 231 cubic inches? Ans. $3351\frac{3}{11}$gal.

WOOD MEASURE.

242. A ***Cord of Wood*** is a pile 8 ft. long, 4 ft. high, and 4ft. wide. It contains 128 *cubic feet*, or 8 *cord feet*.

243. A ***Cord Foot*** is a part of this pile 1 foot long. It is thus 1 foot long, 4 feet wide, and 4 feet high, and contains 16 cubic feet.



It is customary to cut wood, prepared for the market, 4 feet long, and a pile of this wood 8ft. long and 4ft. high is a cord. As many pupils do not live where wood thus prepared can be seen, the teacher can illustrate with sticks 4in. long, showing the *cord* and *cord foot.*

RULE.—*To find the number of cords in a pile of wood, find the number of cubic feet, and reduce to cord feet and cords.*

1. How many cords in a pile of wood 14ft. long, 8ft. wide, and 10ft. high?

SOLUTION.—The number of cubic feet equals $14 \times 8 \times 10 = 1120$ cu. ft.; dividing by 16 to reduce to cord feet, we have 70cd. ft.; dividing 70 by 8 to reduce to cords, we have 8cd. 6cd. ft., or $8\frac{3}{4}$ cords.

EXAMPLES FOR PRACTICE.

2. How many cords in a pile of wood 28ft. long, 12ft. wide, and 8ft. high? Ans. 21 cords.

3. How many cords of wood in a pile 30ft. long, 16ft. wide, and 10ft. high? Ans. $37\frac{1}{2}$ cords.

4. A load of wood containing exactly 1 cord is 4ft. wide and 3ft. high; what is its length? Ans. 10ft. 8in.

5. What is the length of a pile of wood containing 16 cords, if it is 12ft. wide and 8ft. high. Ans. 21ft. 4in.

6. What will be the cost of the wood that can be piled in a shed 20ft. long, 10ft. wide, and 8ft. high, at $4.75 a cord? Ans. 59.37\frac{1}{2}$.

BOARD MEASURE.

244. ***Boards, Planks,*** etc., are usually estimated in *square feet,* called *board feet,* instead of in cubic feet.

245. A ***Board Foot*** is 1 foot long, 1 foot wide, and 1 inch thick. A *cubic foot* is thus equal to 12 *board feet.*

246. Boards whose thickness is less than 1 inch are regarded as an inch thick; in lumber whose thickness is greater than 1 inch, as planks, joists, etc., the thickness is taken into consideration.

Pupils will observe the boards at the lumber-yard, or wherever they may see them, and also obtain correct ideas of planks, joists, scantling, etc.

RULE.—1. *To find the contents of a board, multiply the length in feet by the width in inches, and divide by* 12.

2. *To find the contents of a plank, joist, etc., multiply the length in feet by the width and thickness in inches, and divide by* 12.

NOTE.—When a board tapers regularly, the mean width is used, which is the measure in the middle of the board, or half the width of the two ends.

1. What are the contents of an inch board 16ft. long by 9in. wide?

SOLUTION.—To find the contents, we multiply the length by the width in inches and divide by 12, and we have $16 \times 9 \div 12 = 12$ board feet.

EXAMPLES FOR PRACTICE.

2. Required the contents of a board 21ft. long by 8in. in width. Ans. 14ft.

3. Required the contents of a board 18ft. long, 15in. wide one end and 9in. the other end. Ans. 18ft.

4. How many board feet in a plank 14ft. long, 2 in. thick, and 16in. wide? Ans. $37\frac{1}{3}$ft.

5. How much will 5 joists cost, each 16ft. long by 4in. wide and 3in. thick, at $2\frac{1}{2}$¢ a foot? Ans. $2.00.

6. What cost 20 boards, 12ft. long and 15in. wide, at $2.75 per hundred square feet? Ans. $8.25.

STONE AND MASONRY.

247. ***Masonry*** is sometimes estimated by the *cubic foot*, and sometimes by the *perch*.

248. A ***Perch*** of stone masonry is $16\frac{1}{2}$ft. long by $1\frac{1}{2}$ft. wide and 1ft. high; it thus contains $24\frac{3}{4}$cu. ft.

In estimating *material* for walls, a full allowance is made for doors and windows. In estimating the *labor*, the wall is measured on the *outside*, thus counting the corners twice, and usually only $\frac{1}{2}$ is deducted for openings.

RULE.—*To find the number of perches in a piece of masonry, divide the number of cubic feet by* $24\frac{3}{4}$.

1. How many perches of masonry in a wall 20ft. long, 4ft. high, and 15in. thick?

SOLUTION.—The number of cubic feet equals $20 \times 4 \times 1\frac{1}{4} = 100$, which, divided by $24\frac{3}{4}$, equals $4\frac{4}{99}$ perches.

EXAMPLES FOR PRACTICE.

2. How many perches of masonry in a wall 24ft. long, 6ft. high, and $2\frac{1}{4}$ft. thick? Ans. $13\frac{1}{11}$ perches.

3. What will it cost to dig a cellar 40ft. long, 32ft. wide, and 6ft. 6in. deep, at $6\frac{1}{4}$¢ a cubic foot? Ans. $520.

4. What will the material cost to build a wall in the cellar described in the previous problem, 2ft. 6in. thick, at $1.35 a perch? Ans. $118.77.

5. What will the building of the wall mentioned in the previous problem cost at 55¢ a perch? Ans. $52.

6. What will it cost to fill in a street 600ft. long, 80ft. wide, averaging $5\frac{1}{2}$ft. below grade, at $0.36 a cubic yard. $3520.

BINS AND TANKS.

249. The ***Capacity*** of ***Tanks*** and ***Cisterns*** is usually estimated in *gallons* or *barrels*. A gallon equals 231cu. in., and a barrel equals $31\frac{1}{2}$gal.

250. The ***Capacity*** of ***Bins*** is usually estimated in *bushels*. A bushel equals 2150.42cu. in.

251. Since $\frac{5}{4}$ of 1728cu. in. = 2160cu. in., which is nearly 2150.42cu. in., a bushel equals very nearly $\frac{5}{4}$ of a cubic foot; hence the number of bushels in a bin equals very nearly $\frac{4}{5}$ of the number of cubic feet.

RULE I.—*To find the capacity of a tank, cistern, etc., in gallons, find the contents in cubic inches, and divide by* 231.

RULE II.—*To find the capacity of a bin in bushels, take* $\frac{4}{5}$ *of the number of cubic feet.*

1. How many gallons of water will a tank hold which is 7ft. long, 4ft. wide, and 3ft. deep?

SOLUTION.—The contents equal $7 \times 4 \times 3 = 84$cu. ft., which equals 84×1728cu. in., which, divided by 231, equals $628\frac{4}{11}$ gallons.

EXAMPLES FOR PRACTICE.

2. How many bushels in a bin 8ft. long, 6ft. wide, and 3ft. deep? Ans. $115\frac{1}{5}$bu.

3. How many bushels of corn can be put in a bin 12ft. long, 10ft. wide, and 4ft. deep? Ans. 384bu.

4. How many gallons of water will fill a tank $5\frac{1}{2}$ft. square by $3\frac{1}{2}$ft. deep? Ans. 792gal.

5. How many barrels of water will a tank hold which is 7ft. square by 4ft. deep? Ans. $46\frac{6}{11}$bar.

THE CIRCLE AND CYLINDER.

252. We give the rules for finding the circumference and area of a circle and the contents of a cylinder, which teachers may illustrate with examples if they wish to do so.

RULE I.—*To find the circumference of a circle, multiply the diameter by* $3\frac{1}{7}$.

RULE II.—*To find the diameter of a circle, divide the circumference by* $3\frac{1}{7}$.

RULE III.—*To find the area of a circle, multiply the circumference by one-fourth of the diameter.*

RULE IV.—*To find the contents of a cylinder, find the area of the circle of its base, and multiply the area of the base by the height of the cylinder.*

MISCELLANEOUS PROBLEMS.

(In the Fundamental Rules.)

PROBLEMS IN ADDITION.

1. A man left 850 dollars to his daughter, and 945 dollars to each of his two sons; how much did he leave his two sons? how much did he leave all?

2. Washington was born in the year 1732, Jefferson 11 years after, and Hamilton 15 years after Jefferson; when was Jefferson born? when was Hamilton born?

3. A farmer owns three farms; the first is worth 6560 dollars, the second 385 dollars more, and the third 1387 dollars more than the second; what is the value of the second farm? of the third farm? of all?

4. A has 7586 cents, B has 596 more than A, and C has as many as A and B together; how many has B? how many has C? how many have all?

5. B walked 876 miles, C walked 285 miles more than B, and D walked 985 miles more than C; how far did C walk? how far did D walk? how far did they together walk?

6. A man gave to his wife 4675 dollars, to his son 7582 dollars, to his daughter 3594 dollars, and had 8575 dollars left; what was his fortune?

7. A owns a farm worth 3750 dollars, a wood-lot worth 856 dollars more, and a store worth 987 dollars more than both; what was the value of the wood-lot? of the store? of all three?

8. A man had two sons and three daughters; he gave each son 5896 dollars, and each daughter 4385 dollars; how much did he give to his sons? to his daughters? to all?

9. A butcher sold to one man 876 pounds of meat, to another man 587 pounds more, and to another, 395

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9. A butcher sold to one man 876 pounds of meat, to another man 587 pounds more, and to another, 395

pounds more than both; how much did he sell to the second man? to the third man? how much to all?

10. A raised 3456 bushels of wheat, which was 2475 bushels less than B raised, and D raised 3489 bushels more than both; how much did B raise? how much did D raise? how much did all raise?

11. A bought some land for 8759 dollars, a house for 3768 dollars, and sold them so as to gain 1389 dollars; for what did he sell them?

12. A man bought two lots for 3750 dollars each; and in selling them he gained 278 dollars on the first, and 389 dollars on the second; how much did he gain on both? how much did he receive for both?

13. A has 757 acres of land, B has 285 acres more than A, and C has as many as A and B both; how many acres has B? how many has C? how many have all?

14. William lends his brother 3785 dollars, his sister 4261 dollars, and a friend 485 dollars more than his sister, and has 5858 dollars remaining; how much did he lend his friend, and what was his whole fortune?

PRACTICAL PROBLEMS in Addition and Subtraction.

1. Find the sum of six hundred and five and 18 hundred and ninety-seven.

2. Subtract one thousand and nine from four thousand and seven.

3. Subtract $7567 + 896$ from $4875 + 4736 + 2539$.

4. A had 472 hens, and bought 589 hens, and then sold 985; how many had he then?

5. A farmer had 397 pigs, and bought 85 pigs, and then sold 182 pigs; how many had he then?

6. A drover sold his cows for 2575 dollars, and his sheep for 976 dollars, and gained 594 dollars; how much did he pay for them?

7. A man having 1600 acres of land sold 546 acres

to B, and 289 acres more to C than to B; how much did he sell to C? how much to both? how much remained?

8. Mr. Peters, having 4300 bushels of wheat, sold 1480 bushels, and then bought 1856 bushels more than he sold; how many bushels had he then?

9. Henry had 756 dollars, and his mother gave him enough to make his money 1200 dollars; how much did his mother give him?

10. Sarah bought 575 pins; her mother gave her 289, and her sister gave her enough to make her number 1000; how many did she receive from her sister?

11. Mary's father left her 596 acres of land; she sold 484 acres, and then bought 396 acres; how many acres had she then?

12. A sold 4760 bushels of grain, then sold 1780 bushels, and then had 1875 bushels; how many bushels had he at first?

13. B sold 7560 bushels of rye, then bought 2580 bushels, and then had 5680 bushels; how much did he sell more than he bought? how many bushels had he at first?

14. William had 456 dollars; his father gave him 2528 dollars, he then lost 1869 dollars, and gave away 286 dollars; how many dollars had he then?

15. Three men bought a farm for 20000 dollars; the first paid 7580 dollars, the second paid 6765 dollars, and the third the remainder; how much did the first and second pay? how much did the third pay?

16. A man deposited 8000 dollars in the bank; he drew out at one time 2575 dollars, at another 3467 dollars, at another 1576 dollars; how much remained in the bank?

17. Mr. Bowman, whose property was 35000 dollars, willed 9650 dollars to each of his two sons, 8750 to his daughter, and the remainder to his wife; how much did the children receive? how much did his wife receive?

PRACTICAL PROBLEMS
in Addition, Subtraction, and Multiplication.

1. What is the value of $467 \times 672 - 31675$?

2. What is the value of 672×36 plus 216×42?

3. A man sold his house for 27 times 98 dollars, plus 397 dollars; how much did he receive for it?

4. A sold 24 horses for 168 dollars each, and 63 cows for 34 dollars each; what did he receive for the horses? for the cows? for all?

5. I bought 76 oxen at 68 dollars each, and 327 sheep at 12 dollars each; how much did the oxen cost? how much the sheep? how much did all cost?

6. My barn cost 2318 dollars, my house 3 times as much, and my farm as much as both; what was the cost of the house? the cost of the farm?

7. A man bought 235 cows at 24 dollars each, and sold them for 32 dollars each; how much did he gain by the transaction?

8. A drover bought 78 horses at 164 dollars each, 215 oxen at 59 dollars each, and sold them all for 30000 dollars; how much did he gain?

9. How much must I pay for 7 building-lots, at 2348 dollars each, 5 houses, at 4250 dollars each, and 6 boats, at 3980 dollars each?

10. I bought 78 sheep at 7 dollars a head, and sold them so as to gain 267 dollars; how much did I receive for them?

11. I bought 185 acres of land at 95 dollars an acre, and in selling it I lost 2486 dollars; how much did I receive for it?

12. A speculator bought a farm of 327 acres at 79 dollars an acre, and sold it at 95 dollars an acre; how much did he gain?

13. A man sold his oil stock for 14000 dollars, and then bought a farm containing 93 acres, at 125 dollars

an acre; how much money has he left after paying for it?

14. A clerk receives a salary of 75 dollars a month; he spends 18 dollars a month for board, and 9 dollars for other expenses; how much can he save in 1 month? in 12 months?

15. A farmer having 3420 dollars bought 35 cows at 24 dollars a head, and 36 oxen at 54 dollars a head; how much has he left, after paying for them?

16. Thomas travels 24 miles a day, and Walton travels 52 miles a day; how much farther does Walton travel in 72 days than Thomas?

17. A man bought 336 bushels of potatoes at 65 cents a bushel, and 3 times as many bushels of apples at 98 cents a bushel; what was the entire cost?

18. A's barn cost 1980 dollars, his house 2150 dollars more than the barn, and his farm cost 14 times as much as the barn and house together; what was the cost of the farm?

PRACTICAL PROBLEMS

in Addition, Subtraction, Multiplication, and Division.

1. Divide 42624 by 36 and add 3146 to the quotient.

2. Divide 73305 by 45 and subtract the quotient from 3702.

3. Subtract 3125 from 5213, divide the remainder by 9, and add the quotient to 1745.

4. A man having 18000 dollars leaves his wife 4800 and divides the remainder equally among 6 children; what does each receive?

5. A farm of 24 acres was bought for 4056 dollars and sold at a gain of 3168 dollars; for what was it sold per acre?

6. If 27 men share 11286 dollars equally, how much would each have? how much would A have, if he had four times as much as each, plus 1245 dollars?

7. If 29 men earn 7946 cents in a day, and 25 boys earn 5450 cents in a day, how much more does one man earn in a day than one boy?

8. A horse and 18 oxen are worth 1001 dollars; now, if the horse is worth 245 dollars, what is the value of the oxen? of each ox? Ans. 42 dollars.

9. The value of 3 horses and 15 cows is 1155 dollars if the value of each horse is 225 dollars, what is the value of each cow? Ans. 32 dollars.

10. If you divide 60466 by 49, by what number must I multiply the quotient to produce 9872? Ans. 8.

11. The income of a man who "struck oil" is 400 dollars per day; how many teachers would this employ at a salary of 730 dollars a year?

12. I bought 326 barrels of flour for 2608 dollars, paid 46 dollars for transportation, and sold it at a gain of 280 dollars; what did I receive a barrel?

Ans. 9 dollars.

13. I sold a farm containing 190 acres for 65 dollars an acre, and bought with the proceeds another farm at 95 dollars an acre; how many acres in the latter farm?

14. A drover bought 234 cows at 25 dollars each, and sold 95 of them at cost each; how much must he receive a head for the remainder, to gain 973 dollars?

Ans. 32.

15. If the President of the United States expends 104 dollars daily, how much can he save in a year of 365 days, out of his salary of 50000 dollars?

16. If the Vice-President expends 35 dollars daily, how much can he save at the end of the year, if he has an income of 6450 dollars, besides his salary of 8000 dollars a year?

17. If the Secretary of State expends 16 dollars a day, how much can he save in a year, his salary being 8000 dollars a year and his private income 28 dollars a week?

MISCELLANEOUS PROBLEMS.

1. A has \$4685, B has \$1245 more than A, and C has as much as A and B both; how many dollars has C?

2. C has 438 acres of land, D has 179 acres less, and E has 48 less than C and D together; how many acres have D and E?

3. Henry can walk 30 miles a day, and William can walk 37 miles; how much farther can William walk in 45 days than Henry?

4. Two men start from the same point and walk in opposite directions, one traveling 25, the other 32, miles an hour; how far apart will they be in 148 hours?

5. If a boy can earn \$28 a month, and a man \$47 a month, how much will 6 boys and 9 men earn in a month?

6. If a clerk earns \$150 a month, and spends \$48, how much can he save in 12 months, or a year?

7. A merchant gave \$8.25 a barrel for 96 barrels of flour, and sold it for \$1000; what was the gain?

8. How many bushels of apples can you buy, at \$2½ a bushel, for 56 barrels of flour, at \$7.50 a barrel? Ans. 168.

9. A farmer has 137 hens; now, if he should lay out \$625 for hens, at the rate of 25 cents apiece, how many would he then have? Ans. 2637.

10. If a steamboat goes 14 miles an hour, and a railroad train 32 miles an hour, how far will the steamboat go while the train goes 672 miles? Ans. 294 miles.

11. Mary and Susan had each 1420 cents; after Mary had given Susan 360 and Susan had given Mary 480, how many had each? Ans. Mary, 1540; Susan, 1300.

12. Six men and 8 boys earned a sum of money, and divided it so that each man had \$75 and each boy \$63; how much did they earn?

13. A had 369 acres of land, then bought 720 acres

and then divided the whole into 9 equal farms, and sold 6 of them; how many acres remain?

14. A boy earns $2.50 a day, and pays 75 cents a day for his board; how much can he thus save in a week?

Ans. $9.75.

15. What number must I add to the product of 126 and 72, to make 10000?

16. A lady went to the city with 600 eggs, and sold them at 15 cents a dozen; what did she receive for them?

17. The sum of two numbers is 7809, and one of the numbers is 3725; required the other number, and their difference.

18. The sum of three numbers is 4082; the first number is 1028, the second 2372; required the third number.

19. The difference between two numbers is 709, and one-half of the smaller number equals 482; what is the larger number?

20. A merchant bought 26 bales of cloth, each bale containing 32 pieces, and each piece 24 yards; how many yards did he buy?

21. If a boat sail 378 miles in 9 days, how far can it sail in 15 days at the same rate? Ans. 630 miles.

22. If 28 men can build a lot of wall in 18 days, how many men can build the same wall in 21 days?

Ans. 24 men.

23. A drover bought 365 cows, at $25 a head, and 758 sheep, at $3.50 a head; what was the cost of all?

24. A merchant bought 96 barrels of flour for $960; he sold 58 barrels at $8 a barrel, and the remainder at $12 a barrel; what was the loss?

25. What is the sum of $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{6}$, and $\frac{1}{8}$?

26. Subtract the sum of $\frac{2}{3}$ and $\frac{3}{4}$ from the sum of $\frac{5}{6}$ and $\frac{7}{8}$.

27. Multiply $\frac{3}{4}$ by $\frac{5}{6}$, and add the result to the product of $\frac{4}{5}$ and $\frac{10}{11}$.

28. Multiply $\frac{5}{8}$ by $\frac{8}{9}$, and divide the result by the product of $\frac{7}{8}$ and $\frac{12}{13}$.

29. Find the difference between $\frac{1}{3}$ of $\frac{2}{3}$ of $\frac{3}{4}$ and $\frac{4}{5}$ of $\frac{5}{6}$ of $\frac{6}{7}$.

30. If $\frac{2}{3}$ of a barrel of flour costs \$5.60, what will 4 barrels cost at the same rate? Ans. \$33.60

31. If $\frac{3}{4}$ of a ton of coal is worth \$7.50, what will 12 tons cost at the same rate? Ans. \$120.

32. A man sold $\frac{3}{5}$ of his land for \$9750; what was it all worth at the same rate? Ans. \$16250.

33. Mary lost $\frac{1}{4}$ of her money, and then had \$960 remaining; how much had she before the loss?

34. If $\frac{3}{5}$ of a ton of coal is worth \$4.50, what is $\frac{5}{8}$ of a ton worth at the same rate? Ans. \$9.37$\frac{1}{2}$.

35. If $\frac{3}{4}$ of a lot of land is worth \$234, what is $\frac{5}{6}$ of the same lot worth? Ans. \$260.

36. When rye is worth $\frac{3}{4}$ of a dollar a bushel, how many bushels can be bought for $\frac{7}{8}$ of a dollar? Ans. $\frac{7}{6}$.

37. If one yard of cloth is worth $2\frac{1}{2}$ dollars, how many yards can be bought for \$$5\frac{3}{4}$? Ans. $2\frac{3}{10}$yd.

38. A man owned $\frac{7}{8}$ of a farm, and sold $\frac{1}{3}$ of his share; what part of the whole farm remained? Ans. $\frac{7}{12}$.

39. How many times will 14 gallons fill a vessel that holds $2\frac{1}{3}$ gallons? Ans. 6 times.

40. A father divided 510 acres of land equally among his sons, giving them $63\frac{3}{4}$ acres apiece; how many sons had he?

41. A merchant bought $6\frac{3}{5}$ cords of wood, at \$$6\frac{2}{3}$ a cord, and paid for it with corn, at $\frac{4}{5}$ of a dollar a bushel; how many bushels did it take? Ans. 55bu.

42. If the sum of two fractions is $\frac{5}{6}$, and one of them is $\frac{2}{7}$, what is the other? Ans. $\frac{23}{42}$.

43. If the difference of two fractions is $\frac{3}{10}$, and the smaller is $\frac{4}{15}$, what is the other fraction? Ans. $\frac{17}{30}$.

44. What fraction multiplied by $2\frac{6}{7}$ will give a product of $1\frac{1}{2}$? Ans. $\frac{21}{40}$.

APPENDIX.

THE METRIC SYSTEM

OF WEIGHTS AND MEASURES.

INTRODUCTION.

THE old system of weights and measures in our country is irregular, difficult to learn, and inconvenient to apply. The same is true with the old systems of all nations. Originating by chance, rather than science, they lacked the simplicity of law, and were, therefore, irregular and chaotic.

In 1795, France adopted a system of weights and measures called the Metric System, based upon the decimal method of notation; all the divisions and multiples being by 10. It was regarded as so great an improvement upon the old methods that it has since been adopted by Spain, Belgium, and Portugal, to the exclusion of all other weights and measures, and is in partial use in Holland, Italy, Germany, and Austria, and also in many parts of Spanish America.

In 1864, the British Parliament passed an act permitting its use throughout the empire whenever parties should agree to use it. In 1866, Congress authorized its use in the United States, and provided for its introduction into the post-offices for the weighing of letters and papers.

To facilitate its adoption, a convenient standard of comparison was furnished, by making the new five-cent piece five grams in weight and one fiftieth of a meter, or two centimeters, in diameter. This system will, without doubt, in a few years be in general use in this country.

The *advantages* of the Metric System are numerous and important.

1. It is easily learned; a school-boy can learn it in a single afternoon.

2. It is easily applied, all the operations being the same as in simple numbers.

3. It does away with addition, subtraction, multiplication, division, and reduction of compound numbers and fractions.

4. It will facilitate commerce, giving the nations a universal system of weights and measures.

THE METRIC SYSTEM.

253. The ***Metric System*** of Weights and Measures is based upon the decimal system of notation.

254. In this system we first establish the unit of each measure, and then multiply and divide it by 10.

255. ***Names.***—We first name the *unit* of any measure, and then derive the other denominations by prefixing words to the unit name.

256. The *higher denominations* are expressed by prefixing to the name of the unit,

Deka,	**Hecto,**	**Kilo,**	**Myria.**
10	100	1000	10,000

257. The *lower denominations* are expressed by prefixing to the name of the unit,

Deci,	**Centi,**	**Milli.**
$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$

258. ***Units.***—The following are the different units, with their English pronunciation:—

Measure.	Unit.	Pronunciation.	Measure.	Unit.	Pronunciation.
LENGTH,	**Meter,**	(meter.)	CAPACITY,	**Liter,**	(leeter.)
SURFACE,	**Are,**	(air.)	WEIGHT,	**Gram,**	(gram.)
VOLUME,	**Stere,**	(stair.)	VALUE,	**Dollar.**	

MEASURE OF LENGTH.

259. The ***Meter*** is the *unit of length.* It is the ten-millionth part of the distance from the equator to the poles, and equals 39.37 inches, or 3.28 feet.

TABLE.

10 milli**meters** (m.m.)	equal	1 centi**meter,**	c.m.
10 centi**meters**	"	1 deci**meter,**	d.m.
10 deci**meters**	"	1 **meter,**	M.
10 **meters**	"	1 deka**meter,**	D.M.
10 deka**meters**	"	1 hecto**meter,**	H.M.
10 hecto**meters**	"	1 kilo**meter,**	K.M.
10 kilo**meters**	"	1 myria**meter,**	M.M.

NOTES.—1. The *meter* is very nearly 3 feet, 3 inches, and 3 eighths of an inch in length, which may be easily remembered as the rule of *three threes.*

2. Cloth, etc. are measured by the *meter;* very small distances, by the *millimeter;* great distances, by the *kilometer.*

3. The new 5-cent piece is $\frac{1}{50}$ of a meter in diameter: hence its diameter is $\frac{1}{5}$ of a decimeter, or 2 centimeters.

4. A *decimeter* is about 4 inches; a *kilometer*, about 200 rods, or $\frac{5}{8}$ of a mile; a *millimeter*, about $\frac{1}{25}$ of an inch. The *inch* is about $2\frac{1}{2}$ centimeters; the *foot*, 3 decimeters; the *rod*, 5 meters; the mile, 1600 meters, or 16 hectometers.

QUESTIONS.

1. How many centimeters in a meter?
2. How many millimeters in a meter?
3. How many decimeters in a dekameter?
4. How many meters in a hectometer?
5. How many meters in a kilometer?

MEASURES OF SURFACE.

260. The *Are* is the *unit of surface* used to measure land. The *are* is a *square dekameter.* It equals 119.6 sq. yd., or 0.0247 acre.

TABLE.

10 milli**ares** (m.a.)	equal	1 centi**are**,	c.a.
10 centi**ares**	"	1 deci**are**,	d.a.
10 deci**ares**	"	1 **are**,	A.
10 **ares**	"	1 dek**are**,	D.A.
10 dek**ares**	"	1 hect**are**,	H.A.
10 hect**ares**	"	1 kil**are**,	K.A.
10 kil**ares**	"	1 myri**are**,	M.A.

NOTES.—1. The *are, centiare,* and *hectare* are the denominations principally used, as these are exact squares. The centiare is a square whose side is 1 meter; the hectare is a square whose side is 100 meters.

The are = 100 square meters. The centiare = 1 square meter.
The hectare = 10,000 square meters.

2. The *deciare* is not a *square*, it is merely the tenth of an are; the *dekare* is not a square, it is merely 10 ares.

3. A *hectare* equals very nearly $2\frac{1}{2}$ acres; a *centiare* equals nearly $1\frac{1}{5}$ sq. yd. An *acre* is very nearly 40 ares.

MEASURES OF OTHER SURFACES.

261. All surfaces besides land are measured by the square meter, square decimeter, etc. The measures are shown by the following table:—

TABLE.

100 sq. milli**meters** (m.m.^{2})	= 1 sq. centi**meter**,	c.m.^{2}
100 sq. centi**meters**	= 1 sq. deci**meter**,	d.m.^{2}
100 sq. deci**meters**	= 1 sq. **meter**,	M.^{2}

NOTE.—The measures higher than these are not generally used.

QUESTIONS.

1. How many centiares in an are?
2. How many ares in a hectare?
3. How many square meters in an are?
4. How many square decimeters in an are?
5. How many ares in 640 square meters?

MEASURES OF VOLUME.

262. The *Stere* is the *unit of volume.* It is a *cubic meter,* and equals 35.3166 cubic feet, or 1.308 cu. yd.

TABLE.

10 milli**steres** (m.s.)	equal	1 centi**stere**,	c.s.
10 centi**steres**	"	1 deci**stere**,	d.s.
10 deci**steres**	"	1 **stere**,	S.
10 **steres**	"	1 deka**stere**,	D.S.
10 deka**steres**	"	1 hecto**stere**,	H.S.
10 hecto**steres**	"	1 kilo**stere**,	K.S.
10 kilo**steres**	"	1 myria**stere**,	M.S.

NOTE.—1. Wood is measured by this measure. The *stere, decistere,* and *dekastere* are principally used. 3.6 steres, or 36 decisteres, very nearly equal the common cord.

MEASURES OF OTHER VOLUMES.

263. Other solid bodies are usually measured by the *cubic meter* and its divisions. The measures are shown by the following table.

TABLE.

1000 cubic milli**meters** (**m.m.**^{3})	=1 cubic centi**meter**,	**c.m.**^{3}
1000 cubic centi**meters**	=1 cubic deci**meter**,	**d.m.**^{3}
1000 cubic deci**meters**	=1 cubic **meter**,	**M.**^{3}

NOTE.—The higher denominations are not generally used.

QUESTIONS.

1. How many centisteres in a stere?
2. How many decisteres in a dekastere?
3. How many dekasteres in a kilostere?
4. How many cubic meters in a hectostere?

MEASURES OF CAPACITY.

264. The *Liter* is the *unit of capacity.* It equals a *cubic decimeter;* that is, a cubic vessel whose size is one-tenth of a meter.

This measure is used for measuring liquids and dry substances. The *liter* is a cylinder, and holds 2.1135 pints wine measure, or 1.816 pints dry measure.

TABLE.

10 milli**liters** (**m.l.**)	equal	1 centi**liter**,	**c.l.**
10 centi**liters**	"	1 deci**liter**,	**d.l.**
10 deci**liters**	"	1 **liter**,	**L.**
10 **liters**	"	1 deka**liter**,	**D.L.**
10 deka**liters**	"	1 hecto**liter**,	**H.L.**
10 hecto**liters**	"	1 kilo**liter**,	**K.L.**
10 kilo**liters**	"	1 myria**liter**,	**M.L.**

NOTES.—1. The *liter* is principally used in measuring *liquids*, and the *hectoliter* in measuring grains, etc.

2. The *liter* equals nearly $1\frac{1}{18}$ liquid quarts, or $\frac{9}{10}$ of a dry quart, or nearly $\frac{1}{36}$ of a bushel measure.

3. The *hectoliter* is about $2\frac{5}{8}$ bushels, or $\frac{2}{3}$ of a barrel. 4 liters are a little more than a *gallon;* 35 liters, very nearly a *bushel.*

QUESTIONS.

1. How many liters in a hectoliter?
2. How many liters in a kiloliter?
3. How many deciliters in a dekaliter?
4. How many liters in a cubic meter? Ans. 1000.
5. How many liters in a stere? Ans 1000.

MEASURE OF WEIGHT.

265. The ***Gram*** is the *unit of weight.* It is the weight of a cubic centimeter of distilled water at the temperature of melting ice. The gram equals 15.432 troy grains.

TABLE.

10 milli**grams** (**m.g.**)	equal	1 centi**gram,**	**c.g.**
10 centi**grams**	"	1 deçi**gram,**	**d.g.**
10 deci**grams**	"	1 **gram,**	**G.**
10 **grams**	"	1 deka**gram,**	**D.G.**
10 deka**grams**	"	1 hecto**gram,**	**H.G.**
10 hecto**grams**	"	1 kilo**gram,**	**K.G., or K.**
10 kilo**grams**	"	1 myria**gram,**	**M.G.**

NOTES.—1. The *gram* is used in weighing letters, in mixing and compounding medicines, and in weighing all very light articles. The new 5-cent coin (dated 1866) weighs 5 grams.

2. The *kilogram* is the ordinary unit of weight, and is generally abbreviated into *kilo.* It equals about $2\frac{1}{5}$ pounds avoirdupois. Meat, sugar, etc. are bought and sold by the *kilogram.*

3. In weighing heavy articles, two other weights, the *quintal* (100 kilograms) and the *tonneau* (1000 kilograms), are used. The *tonneau* is between our *short ton* and *long ton.*

4. The *avoirdupois ounce* is about 28 *grams;* the *pound* is a little less than $\frac{1}{2}$ a *kilo.*

QUESTIONS.

1. How many grams in a kilogram?
2. How many milligrams in a gram?
3. How many decigrams in a kilogram?
4. How many hectograms in a myriagram?

ADDITION TABLE.

2 and	3 and	4 and	5 and
0 are 2	0 are 3	0 are 4	0 are 5
1 " 3	1 " 4	1 " 5	1 " 6
2 " 4	2 " 5	2 " 6	2 " 7
3 " 5	3 " 6	3 " 7	3 " 8
4 " 6	4 " 7	4 " 8	4 " 9
5 " 7	5 " 8	5 " 9	5 " 10
6 " 8	6 " 9	6 " 10	6 " 11
7 " 9	7 " 10	7 " 11	7 " 12
8 " 10	8 " 11	8 " 12	8 " 13
9 " 11	9 " 12	9 " 13	9 " 14
10 " 12	10 " 13	10 " 14	10 " 15
11 " 13	11 " 14	11 " 15	11 " 16
12 " 14	12 " 15	12 " 16	12 " 17

6 and	7 and	8 and	9 and
0 are 6	0 are 7	0 are 8	0 are 9
1 " 7	1 " 8	1 " 9	1 " 10
2 " 8	2 " 9	2 " 10	2 " 11
3 " 9	3 " 10	3 " 11	3 " 12
4 " 10	4 " 11	4 " 12	4 " 13
5 " 11	5 " 12	5 " 13	5 " 14
6 " 12	6 " 13	6 " 14	6 " 15
7 " 13	7 " 14	7 " 15	7 " 16
8 " 14	8 " 15	8 " 16	8 " 17
9 " 15	9 " 16	9 " 17	9 " 18
10 " 16	10 " 17	10 " 18	10 " 19
11 " 17	11 " 18	11 " 19	11 " 20
12 " 18	12 " 19	12 " 20	12 " 21

10 and	11 and	12 and	13 and
0 are 10	0 are 11	0 are 12	0 are 13
1 " 11	1 " 12	1 " 13	1 " 14
2 " 12	2 " 13	2 " 14	2 " 15
3 " 13	3 " 14	3 " 15	3 " 16
4 " 14	4 " 15	4 " 16	4 " 17
5 " 15	5 " 16	5 " 17	5 " 18
6 " 16	6 " 17	6 " 18	6 " 19
7 " 17	7 " 18	7 " 19	7 " 20
8 " 18	8 " 19	8 " 20	8 " 21
9 " 19	9 " 20	9 " 21	9 " 22
10 " 20	10 " 21	10 " 22	10 " 23
11 " 21	11 " 22	11 " 23	11 " 24
12 " 22	12 " 23	12 " 24	12 " 25

NOTE.—Pupils may be taught to derive the *elementary differences* from the *elementary sums*.

MULTIPLICATION TABLE.

Once	**2 times**	**3 times**	**4 times**
1 is 1	1 are 2	1 are 3	1 are 4
2 " 2	2 " 4	2 " 6	2 " 8
3 " 3	3 " 6	3 " 9	3 " 12
4 " 4	4 " 8	4 " 12	4 " 16
5 " 5	5 " 10	5 " 15	5 " 20
6 " 6	6 " 12	6 " 18	6 " 24
7 " 7	7 " 14	7 " 21	7 " 28
8 " 8	8 " 16	8 " 24	8 " 32
9 " 9	9 " 18	9 " 27	9 " 36
10 " 10	10 " 20	10 " 30	10 " 40
11 " 11	11 " 22	11 " 33	11 " 44
12 " 12	12 " 24	12 " 36	12 " 48
5 times	**6 times**	**7 times**	**8 times**
1 are 5	1 are 6	1 are 7	1 are 8
2 " 10	2 " 12	2 " 14	2 " 16
3 " 15	3 " 18	3 " 21	3 " 24
4 " 20	4 " 24	4 " 28	4 " 32
5 " 25	5 " 30	5 " 35	5 " 40
6 " 30	6 " 36	6 " 42	6 " 48
7 " 35	7 " 42	7 " 49	7 " 56
8 " 40	8 " 48	8 " 56	8 " 64
9 " 45	9 " 54	9 " 63	9 " 72
10 " 50	10 " 60	10 " 70	10 " 80
11 " 55	11 " 66	11 " 77	11 " 88
12 " 60	12 " 72	12 " 84	12 " 96
9 times	**10 times**	**11 times**	**12 times**
1 are 9	1 are 10	1 are 11	1 are 12
2 " 18	2 " 20	2 " 22	2 " 24
3 " 27	3 " 30	3 " 33	3 " 36
4 " 36	4 " 40	4 " 44	4 " 48
5 " 45	5 " 50	5 " 55	5 " 60
6 " 54	6 " 60	6 " 66	6 " 72
7 " 63	7 " 70	7 " 77	7 " 84
8 " 72	8 " 80	8 " 88	8 " 96
9 " 81	9 " 90	9 " 99	9 " 108
10 " 90	10 " 100	10 " 110	10 " 120
11 " 99	11 " 110	11 " 121	11 " 132
12 " 108	12 " 120	12 " 132	12 " 144

NOTE.—Pupils may be taught to derive the *elementary quotients* from the *elementary products.*

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